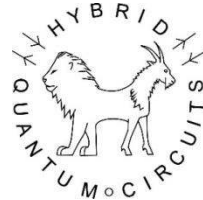




laboratoire pierre aigrain
électronique et photonique quantiques



Mesoscopic Quantum Electrodynamics with carbon nanotubes



LPA – Ecole Normale Supérieure – Paris

Experiments:

Laure Bruhat

T. Cubaynes

J.J. Viennot

M.C. Dartiailh

M.M. Desjardins

T. Kontos

Theory:

A. Cottet

B. Douçot



What is mesoscopic QED ?

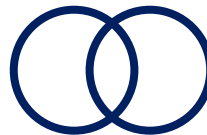
Mesoscopic quantum electrodynamics

=

Cavity quantum electrodynamics with mesoscopic circuits

Here : Quantum dot circuits

Cavity QED



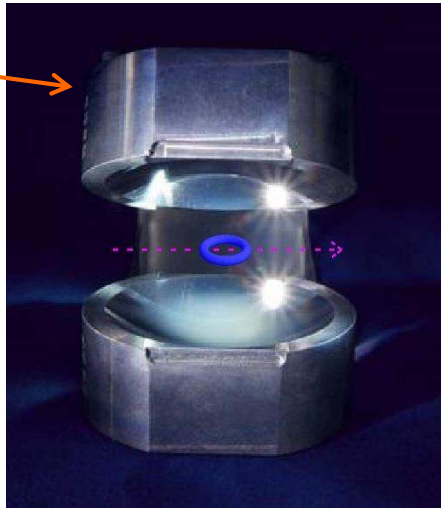
Quantum transport

Cavity Quantum ElectroDynamics (QED): from optical systems to superconducting chips

Atomic Cavity QED

High-
quality
mirror
cavity

Rydberg
atom

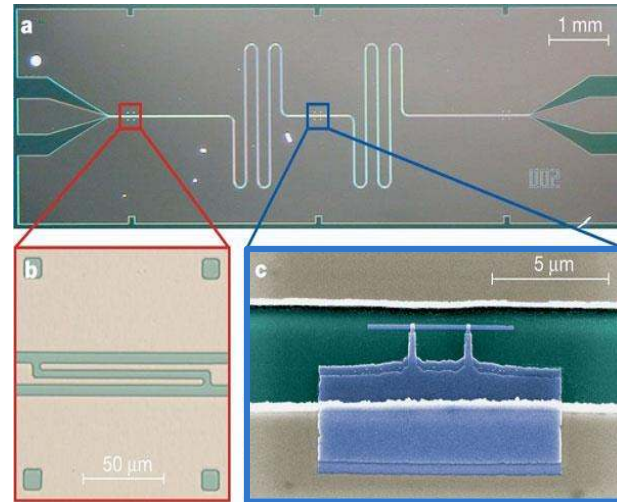


M. Brune et al., PRL 76, 1800 (1996)

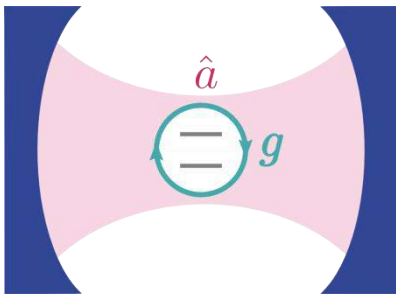
Circuit QED

Coplanar
waveguide
cavity

Super-
conducting
qubit



A. Wallraff et, Nature 431, 162 (2004)



Jaynes-Cummings Hamiltonian

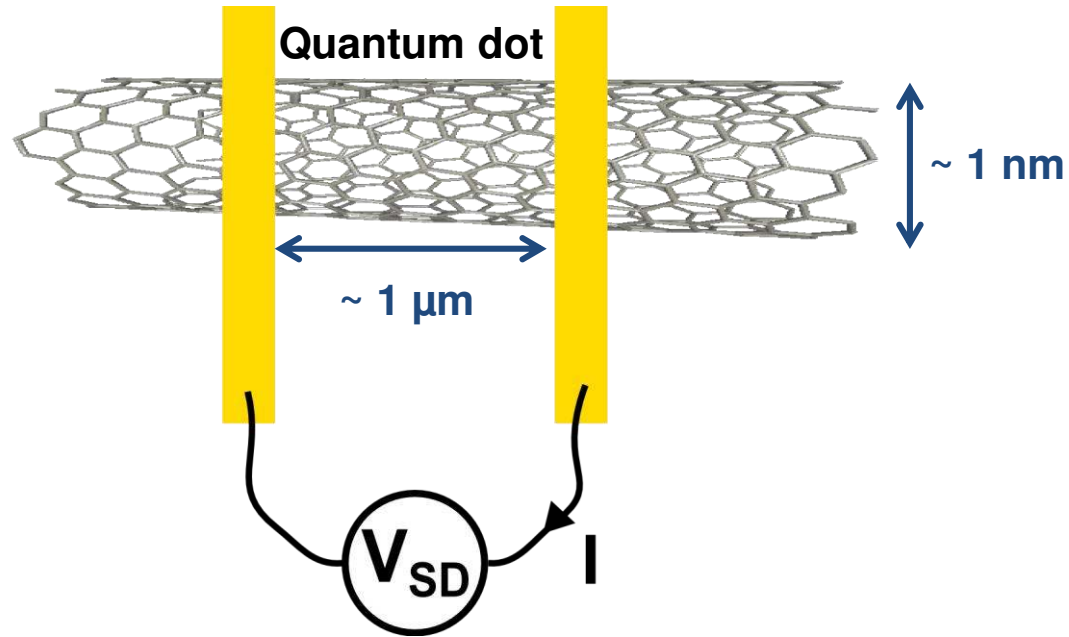
$$\hat{H} = -\frac{\hbar\omega_s}{2}\hat{\sigma}_z + \hbar\omega_{cav}\hat{a}^\dagger\hat{a} + g\hat{\sigma}_x(\hat{a}^\dagger + \hat{a})$$

Two level
system

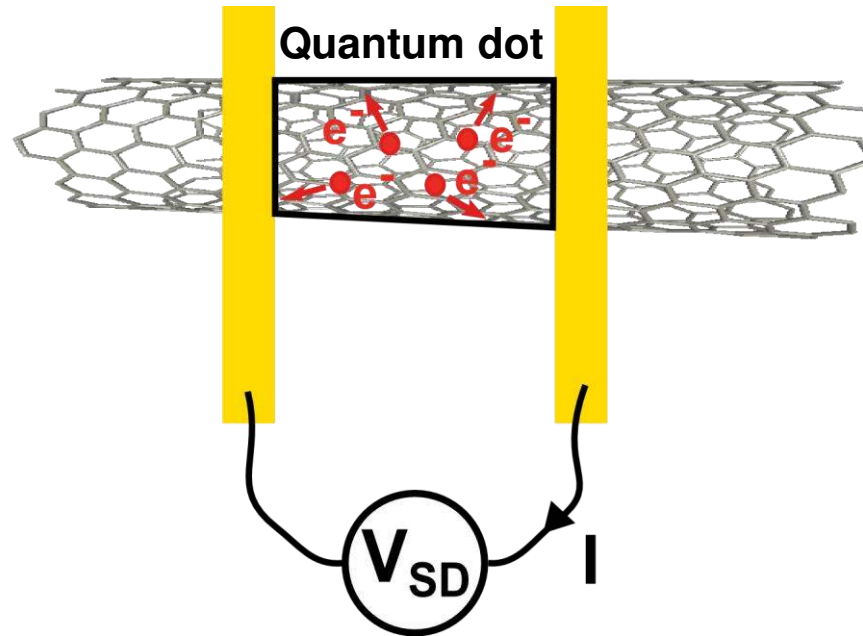
Cavity
photons

Transverse
coupling

Carbon nanotube quantum dot circuits



Carbon nanotube quantum dot circuits

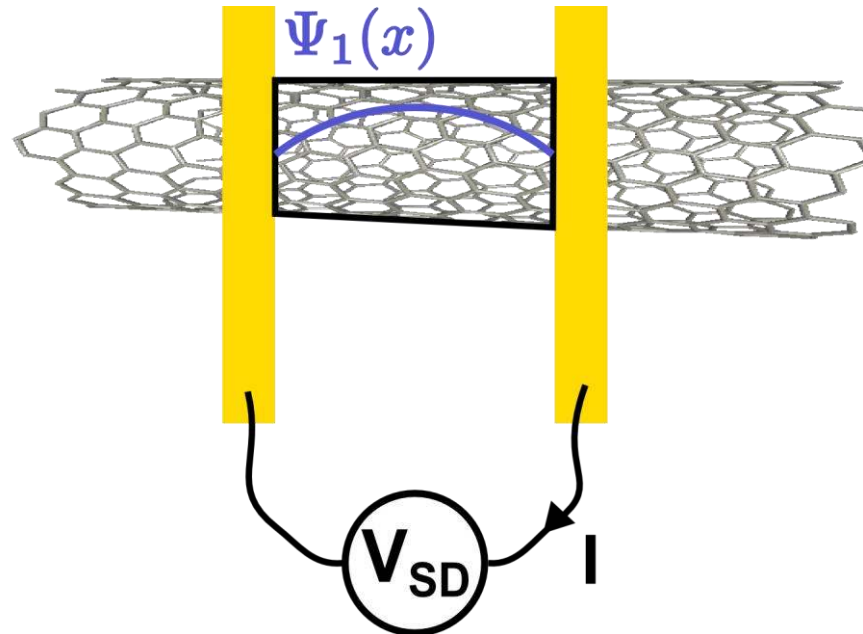


Repulsion between electrons

U : Coulomb energy

Typical range 1-10 meV

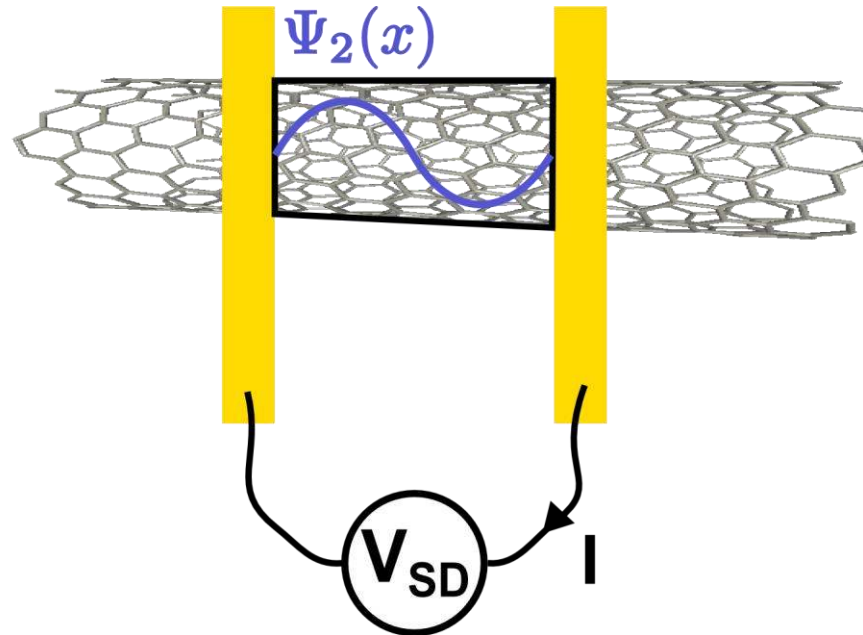
Carbon nanotube quantum dot circuits



Repulsion between electrons
U : Coulomb energy
 Typical range 1-10 meV

Confinement
 ΔE : level spacing
 Typically ~ 1 meV

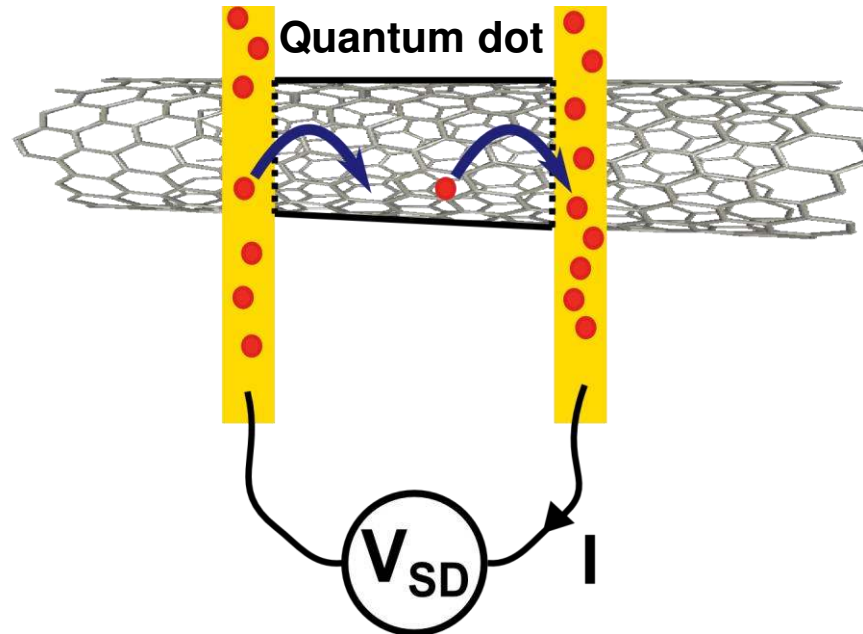
Carbon nanotube quantum dot circuits



Confinement
 ΔE : level spacing
 Typically ~ 1 meV

Repulsion between electrons
 U : Coulomb energy
 Typical range 1-10 meV

Carbon nanotube quantum dot circuits

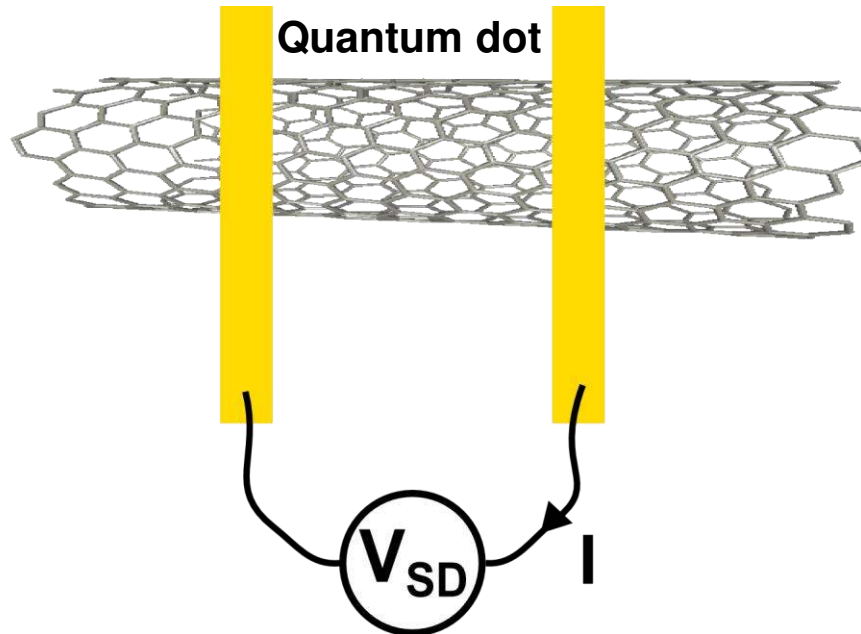


Repulsion between electrons
 U : Coulomb energy
 Typical range 1-10 meV

Confinement
 ΔE : level spacing
 Typically ~ 1 meV

Electron transfer dot-contacts
 Γ : tunnelling energy
 Typical range μeV -meV

Carbon nanotube quantum dot circuits



3 characteristic energy scales + **thermal energy $k_B T$**

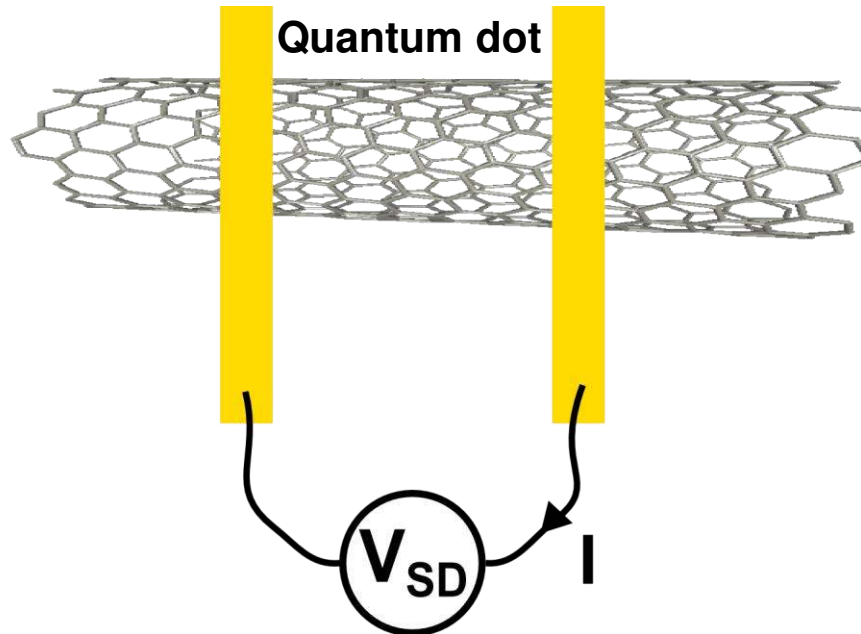
Room temperature $\approx 20^\circ\text{C} \sim 25 \text{ meV}$

Repulsion between electrons
 U : Coulomb energy
 Typical range 1-10 meV

Confinement
 ΔE : level spacing
 Typically $\sim 1 \text{ meV}$

Electron transfer dot-contacts
 Γ : tunnelling energy
 Typical range μeV -meV

Carbon nanotube quantum dot circuits



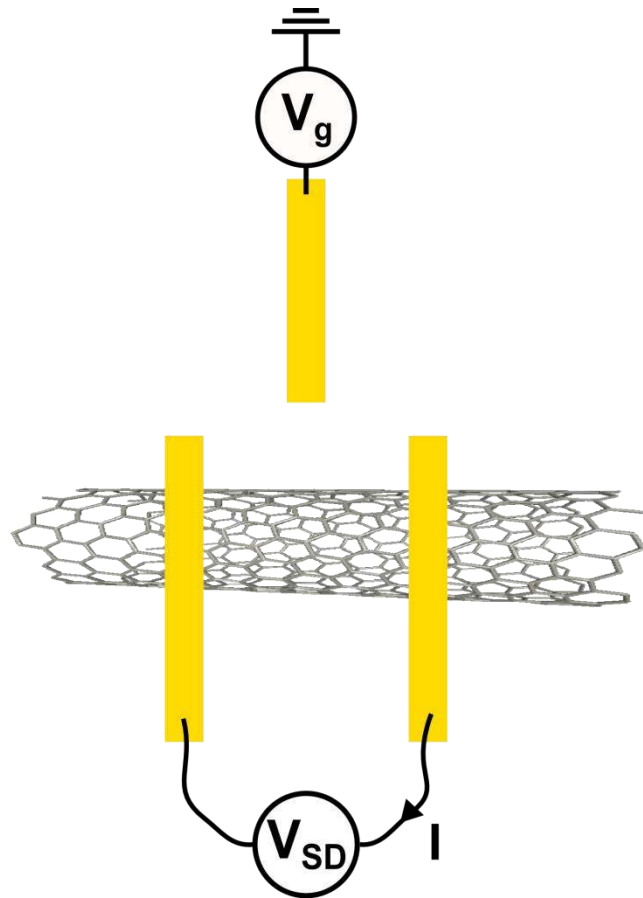
3 characteristic energy scales + **thermal energy $k_B T$**

$50\text{mK} \approx 4 \mu\text{eV} !$

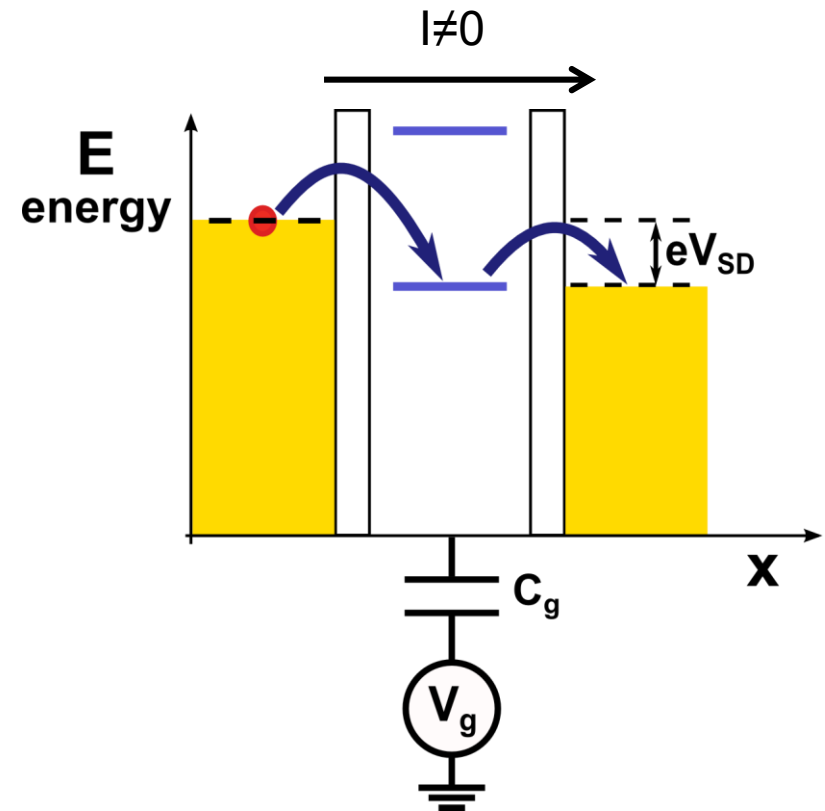
Repulsion between electrons
U : Coulomb energy
 Typical range 1-10 meV

Confinement
 ΔE : level spacing
 Typically ~ 1 meV

Electron transfer dot-contacts
 Γ : tunnelling energy
 Typical range μeV -meV



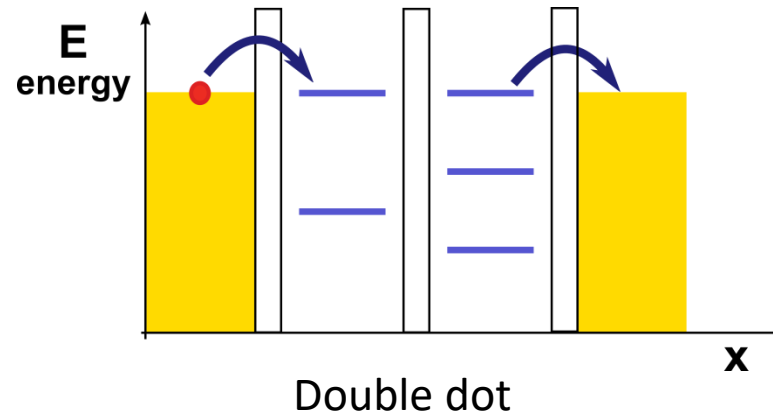
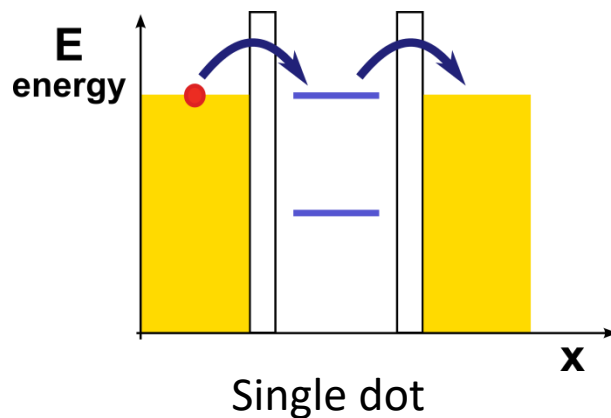
Circuit energy profile



2 electrical knobs : gate voltage V_g and bias voltage V_{SD}

From single dot to multiple dot circuits

Nanofabrication allows to complexify the **circuit geometry**.



Triple dot
Quadruple dot...

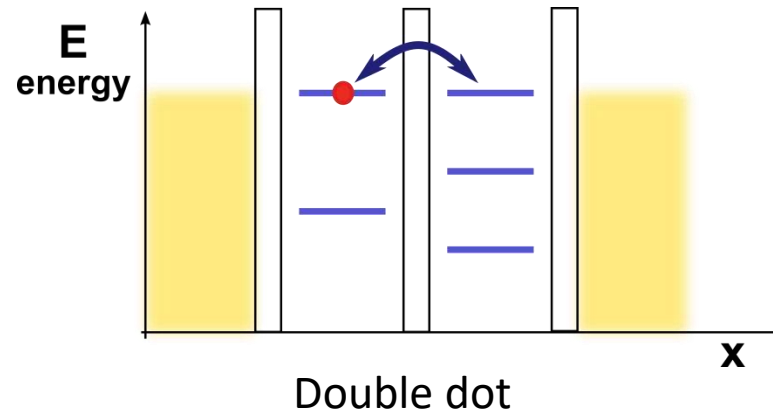
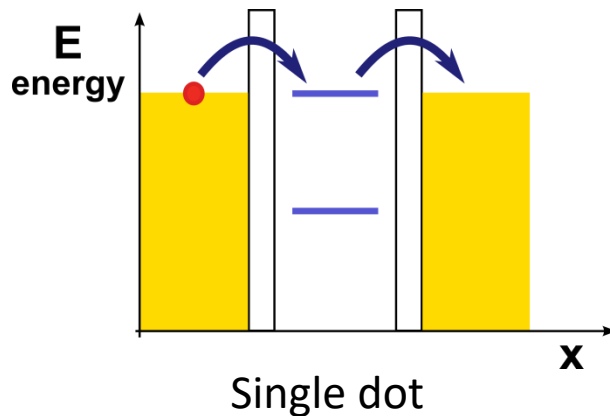
What can we do with these circuits ?

Study **quantum transport**

- Involves coupling to **continua of states**.
- Possible **many-body physics**

From single dot to multiple dot circuits

Nanofabrication allows to complexify the **circuit geometry**.



Triple dot
Quadruple dot...



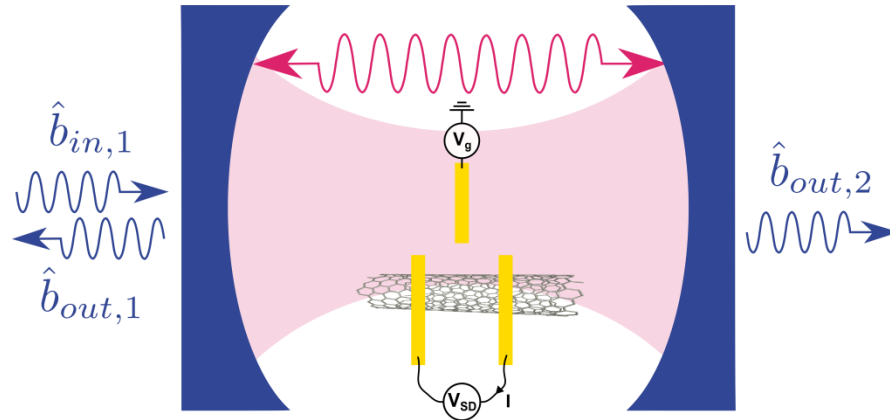
Study **quantum transport**

- Involves coupling to **continua of states**.
- Possible **many-body physics**

Study **atomic-like physics**

- Internal dynamics of artificial molecules « isolated » from the contacts
- Possible to make qubits for **quantum information**

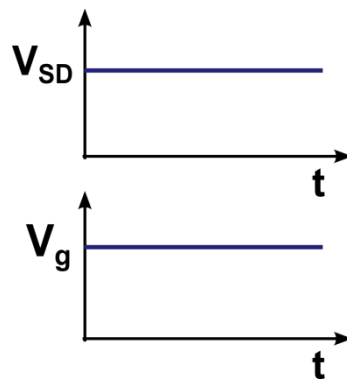
Interests of mesoscopic QED



WHY ???

✓ DC transport

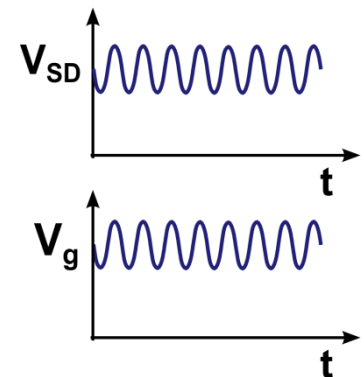
Constant voltages



Many experiments for more than 20 years

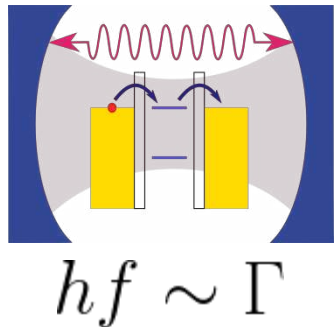
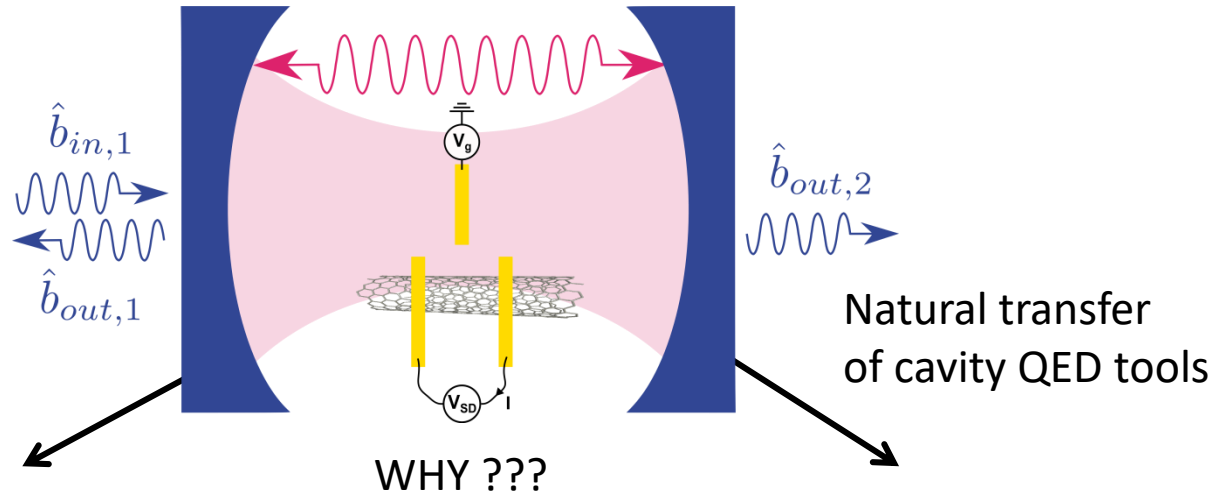
AC transport ?

Time-dependent voltages

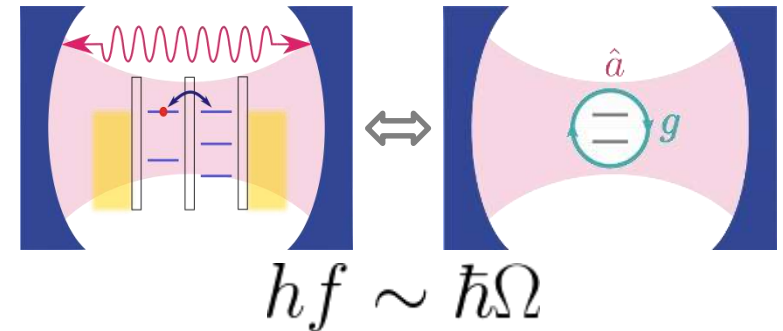


Fewer experiments

Interests of mesoscopic QED



$hf_c \sim$ typical energy scale in the circuit



Open quantum dots :

Study quantum transport in a different way ?

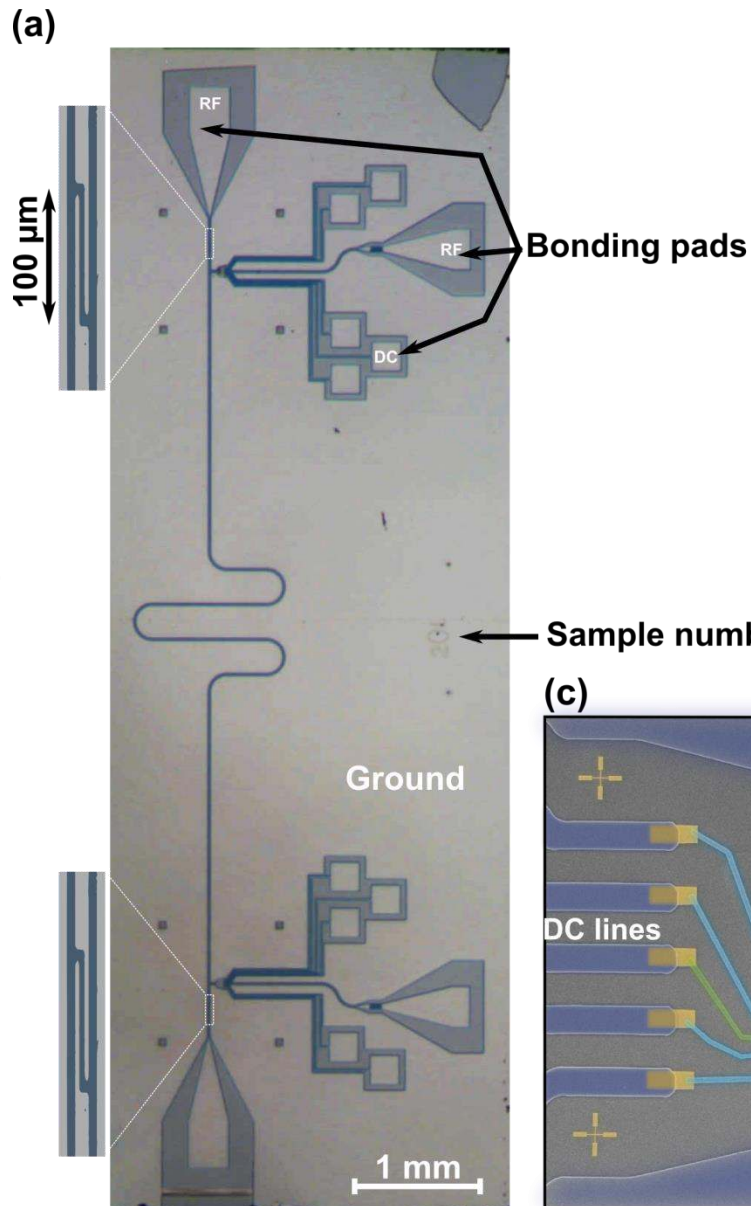
See [Exp I](#) : S-dot-N in cavity

Closed quantum dots:

Probe internal quantum states of a mesoscopic circuit ?

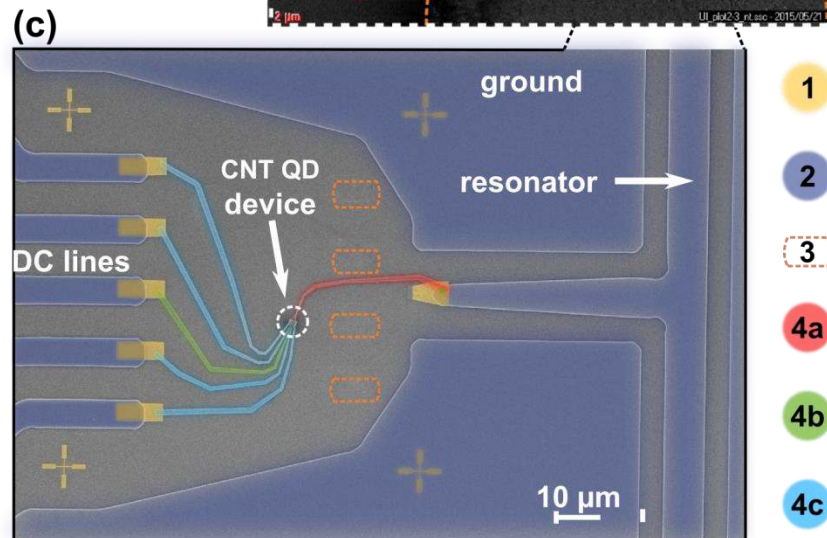
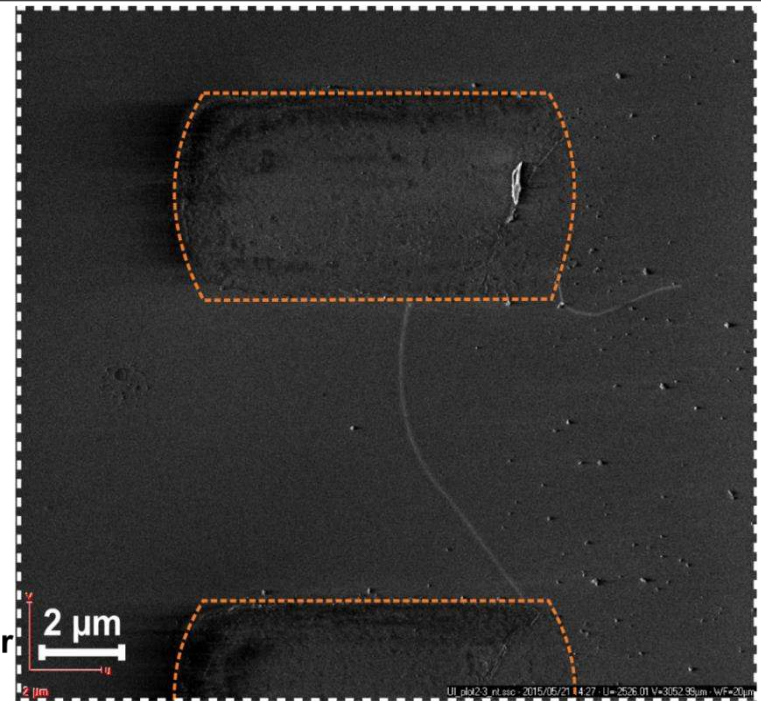
See [Exp II](#) : Superconductor-double quantum dot in cavity

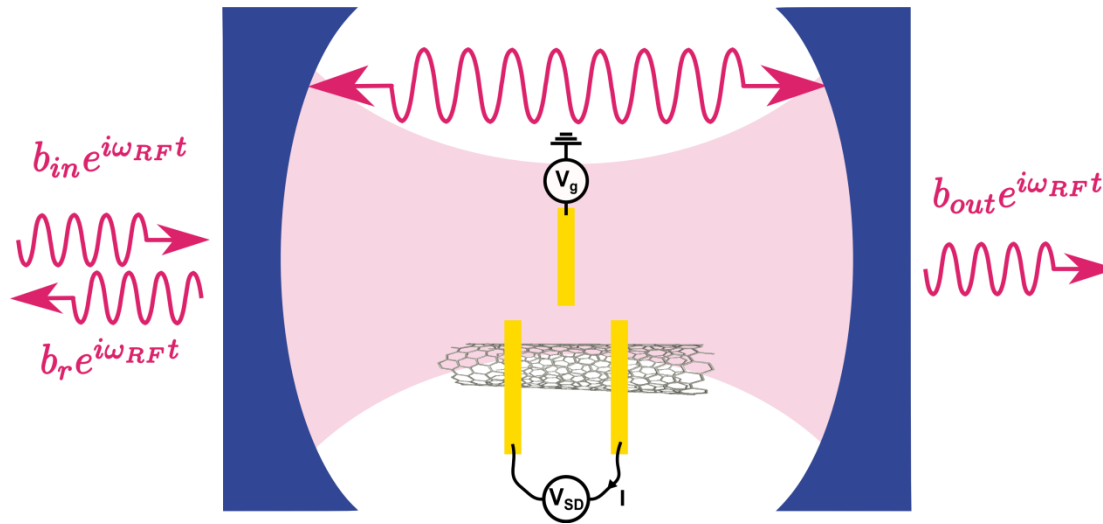
Sample in real life...



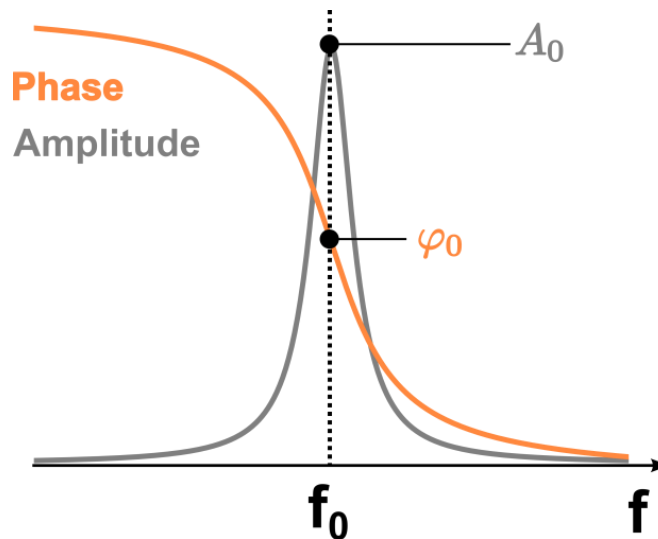
Resonance frequency
~6 GHz

Sample number





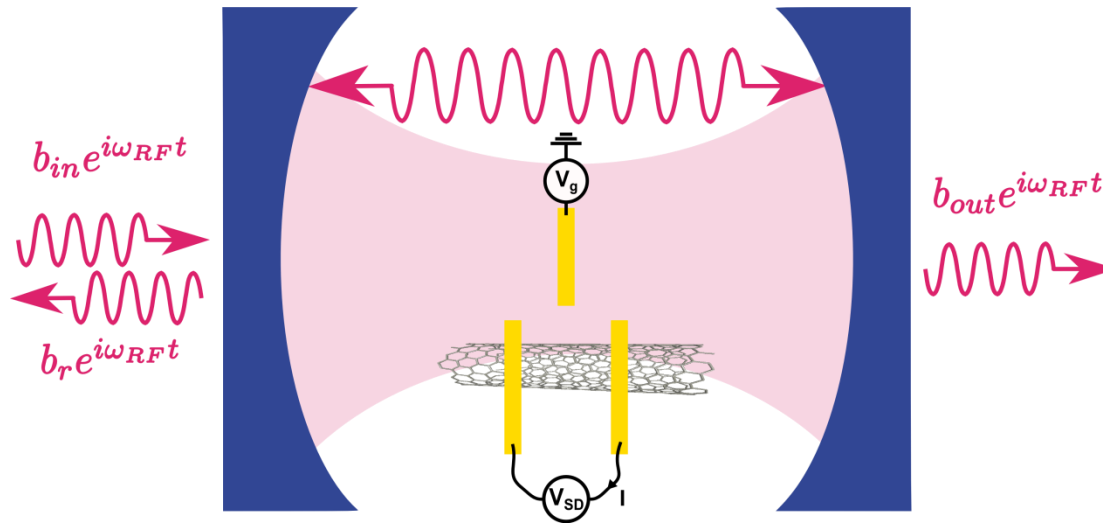
**Cavity-current
crossed characterisation:
Current I
+
Microwave transmitted signal**



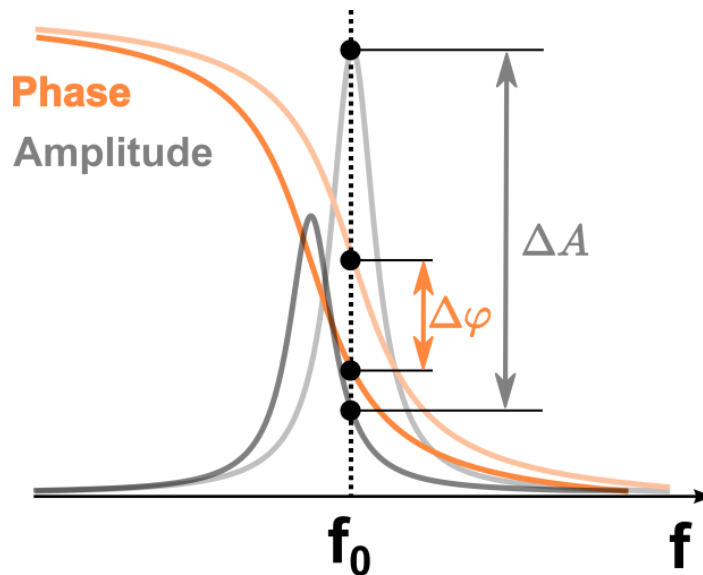
$$\frac{b_t}{b_{in}} = A_0 e^{i\varphi_0}$$

Bare cavity resonance :

- Resonance frequency f_0
- Linewidth related to sensitivity



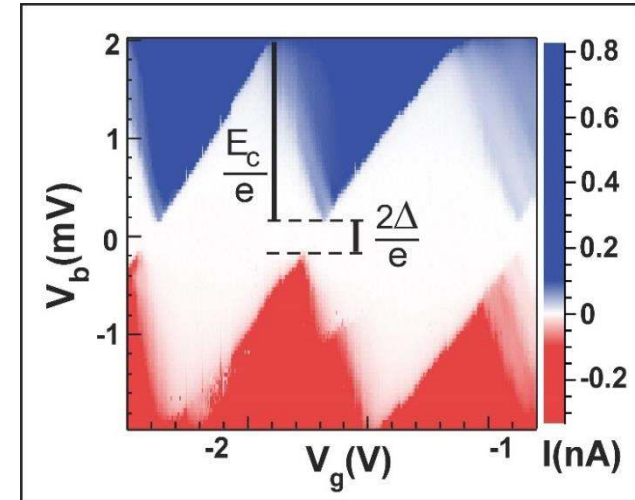
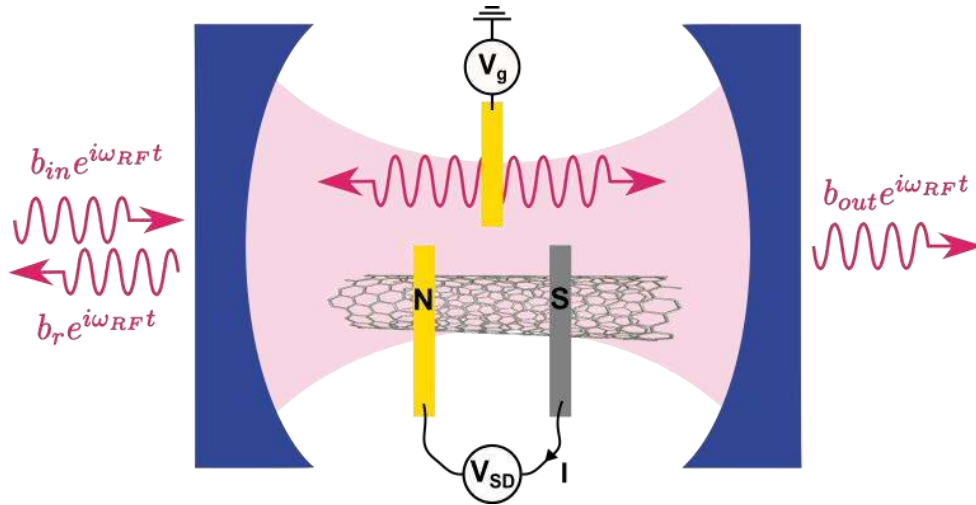
**Cavity-current
crossed characterisation:
Current I
+
Microwave transmitted signal**



$$\frac{b_t}{b_{in}} = (A_0 + \Delta A) e^{i(\varphi_0 + \Delta\varphi)}$$

Usual measurement :
**Amplitude and phase shift
occasionned by the circuit at
the bare cavity frequency f_0**

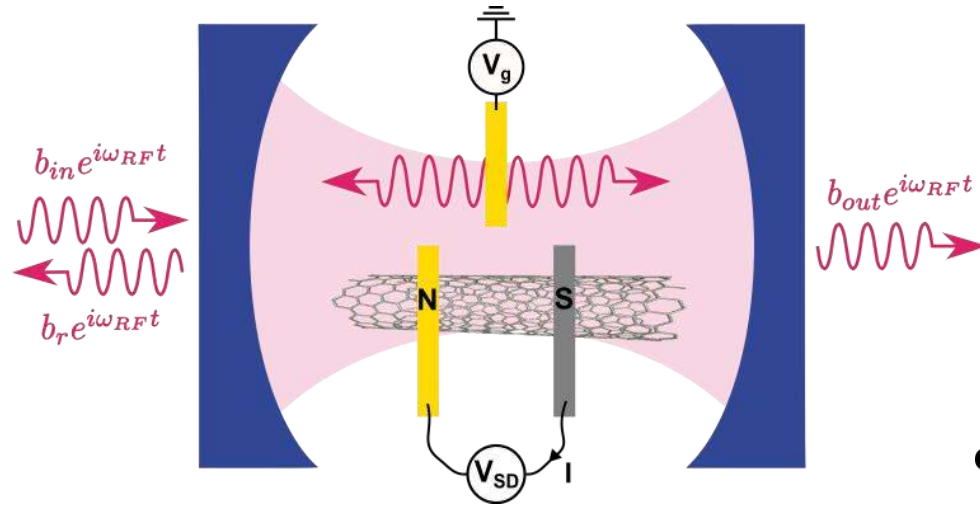
Exp I: Dot contacted to normal (N) and superconducting (S) reservoirs



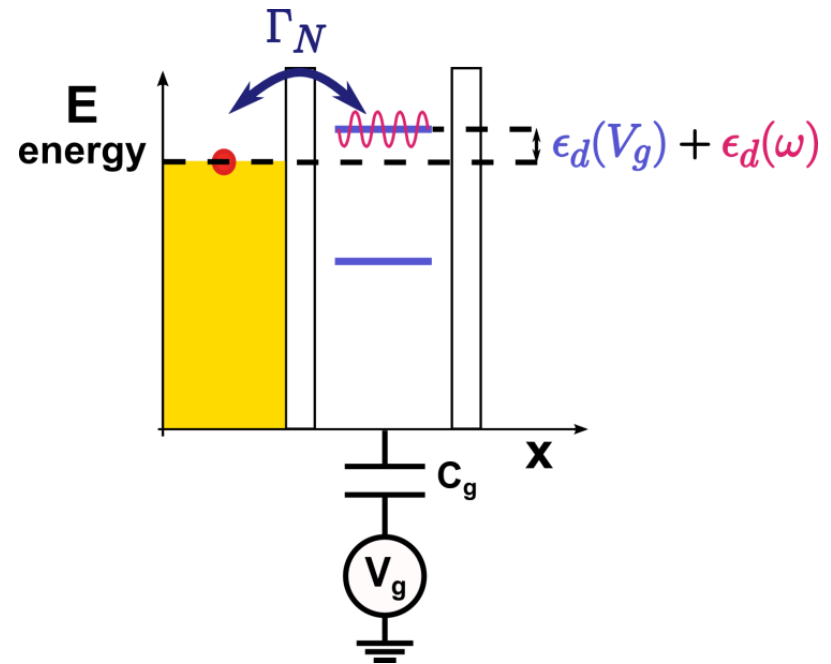
- Coulomb diamonds = **interacting electrons**
- No current in gap = **no Cooper pair transport**

What does this imply ?

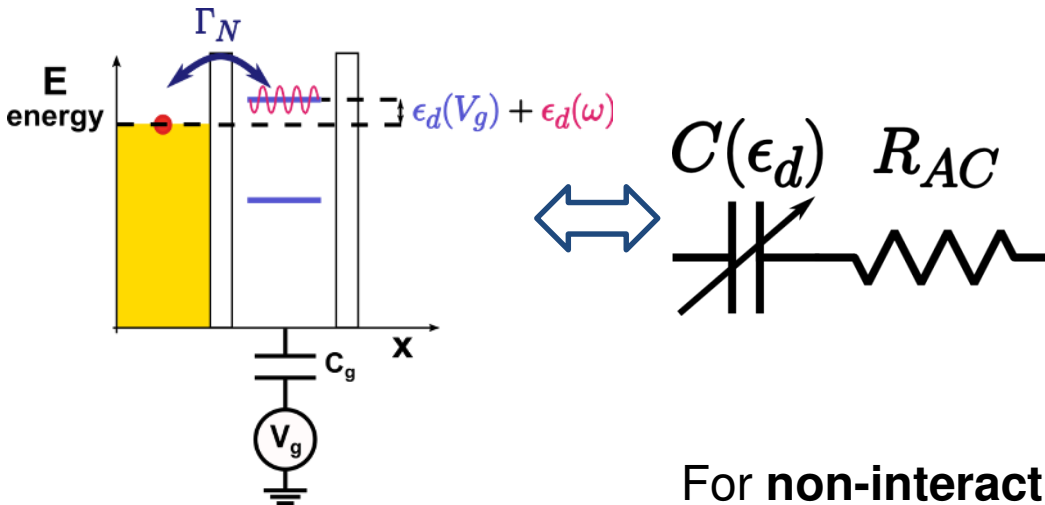
Dot contacted to normal (N) and superconducting (S) reservoirs



$\Rightarrow$ At $V_b=0$, effective dot/N junction



Simplest situation to study tunnelling dynamics !



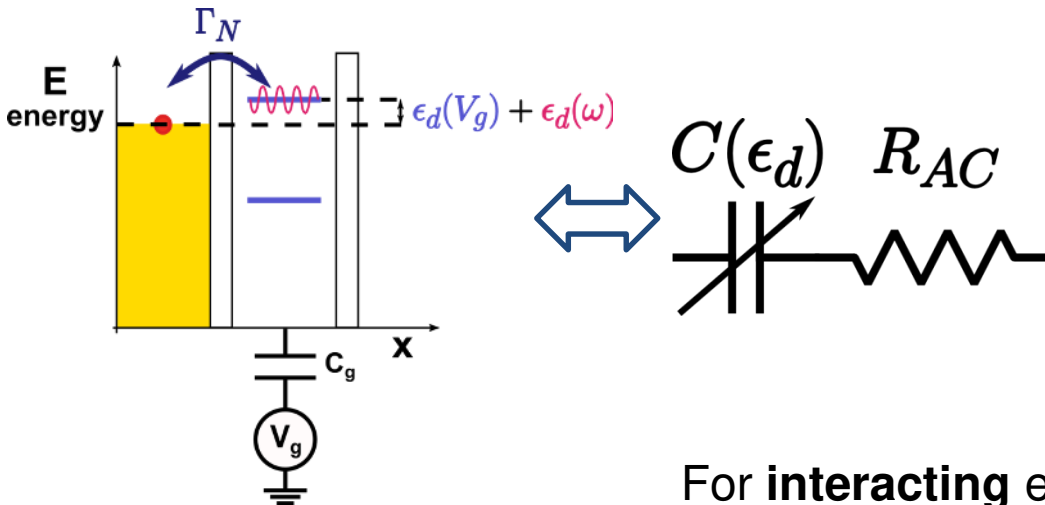
Assuming :

- large tunnel rate $\Gamma_N \gg \omega$

For **non-interacting** electrons:

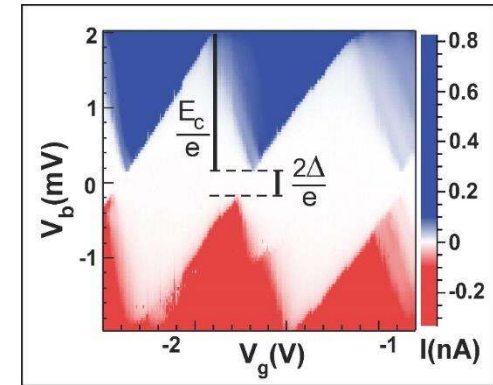
Universality of R_{AC}	Experiment	Theory
$R_{AC} = h/(N \cdot 2e^2)$ with N number of channels	<i>J. Gabelli et al., Science 313, 499 (2006)</i>	<i>M. Büttiker et al., Phys. Lett. A 180, 364 (1993)</i>
R_{AC} independent of dot energy ϵ_d	?	<i>M. Büttiker et al., Phys. Lett. A 180, 364 (1993)</i>

Charge relaxation in the interacting case



Assuming :

- large tunnel rate $\Gamma_N \gg \omega$



For **interacting** electrons:

Universality of R_{AC}	Experiment	Theory
$R_{AC} = h/(N \cdot 2e^2)$ with N number of channels	?	<i>C. Mora and K. Le Hur, Nat. Phys. 6, 697 (2010).</i>
R_{AC} independent of dot energy ϵ_d	?	<i>C. Mora and K. Le Hur, Nat. Phys. 6, 697 (2010).</i>

Can we learn something with our setup ?

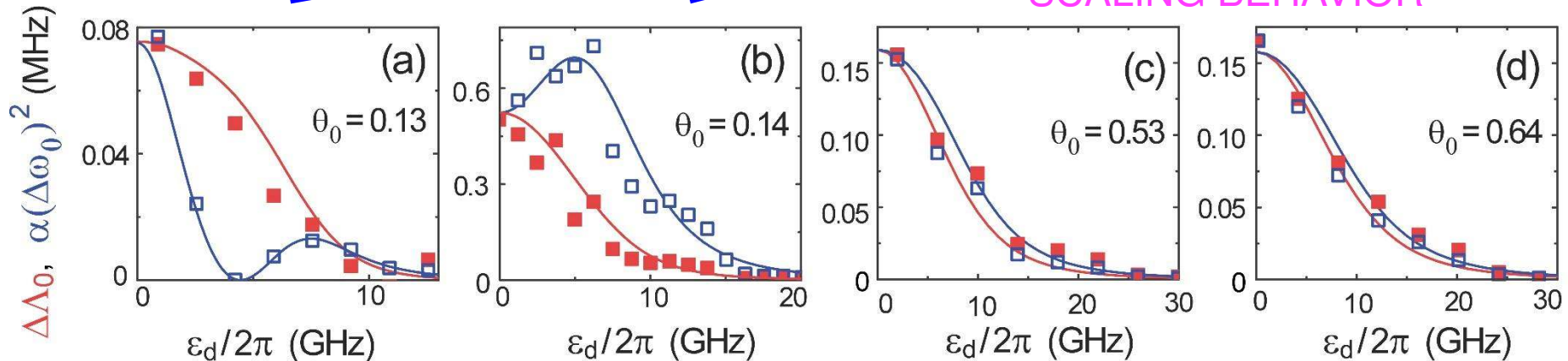
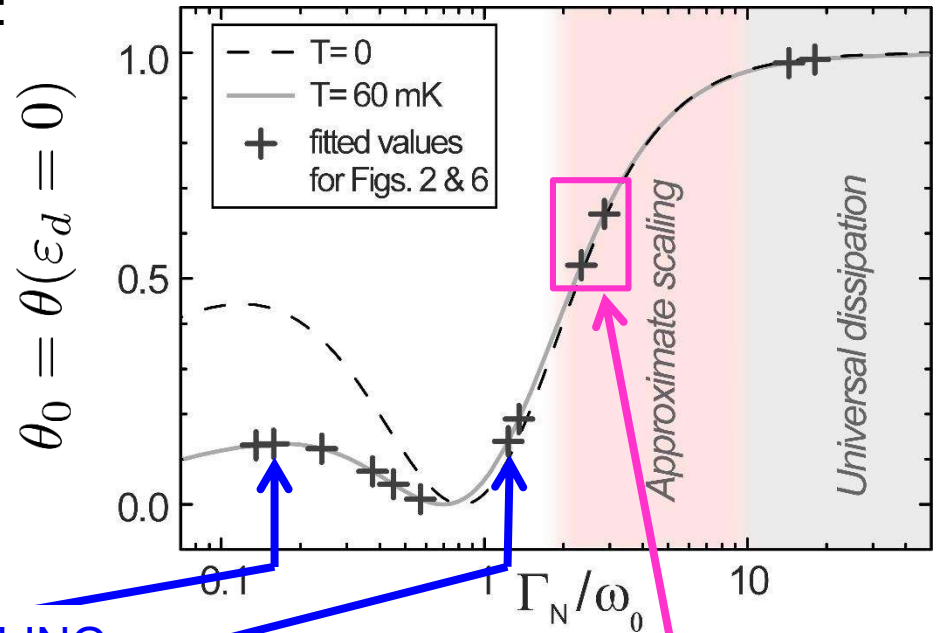
Towards universal charge dissipation

Universality predictions with 2 channels :

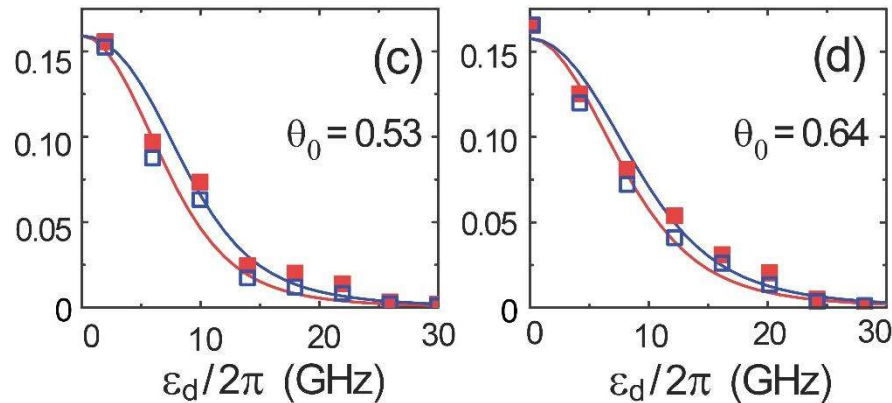
$$R_{AC} = \frac{h}{4e^2 \theta(\varepsilon_d)}$$

$$\lim_{\Gamma_N / \omega_0 \rightarrow +\infty} \theta(\varepsilon_d) = 1$$

$$\theta(\varepsilon_d) = \frac{\pi}{2} \frac{\omega_0}{g^2} \frac{(\Delta\omega_0)^2}{\Delta\Lambda_0}$$



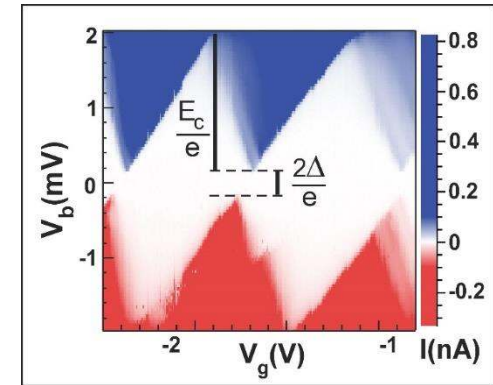
SCALING BEHAVIOR



For interacting electrons:

Assuming :

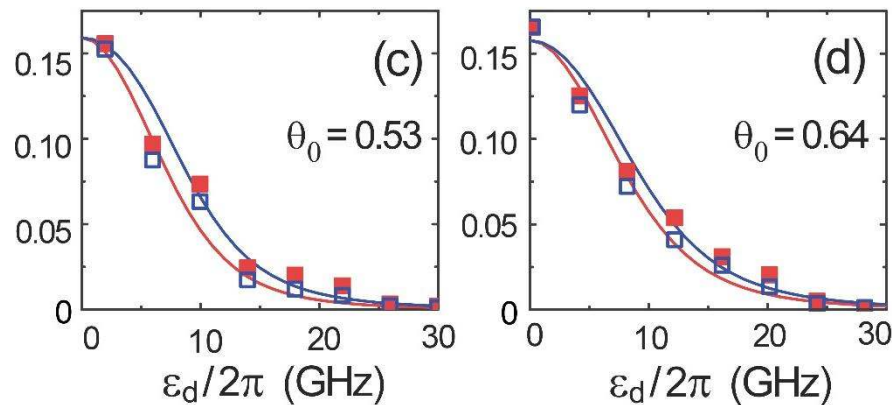
- large tunnel rate $\Gamma_N \gg \omega$



Universality of R_{AC}	Experiment	Theory
$R_{AC} = h/(N \cdot 2e^2)$ with N number of channels	?	<i>M. Filippone, et al., Phys. Rev. Lett. 107, 176601 (2011)</i>
R_{AC} independent of dot energy ε_d	<i>LE. Bruhat et al, PRX 2016</i>	<i>M. Lee, et al., Phys. Rev. B 83, 201304 (2011)</i>

We put one piece in the big puzzle of science!

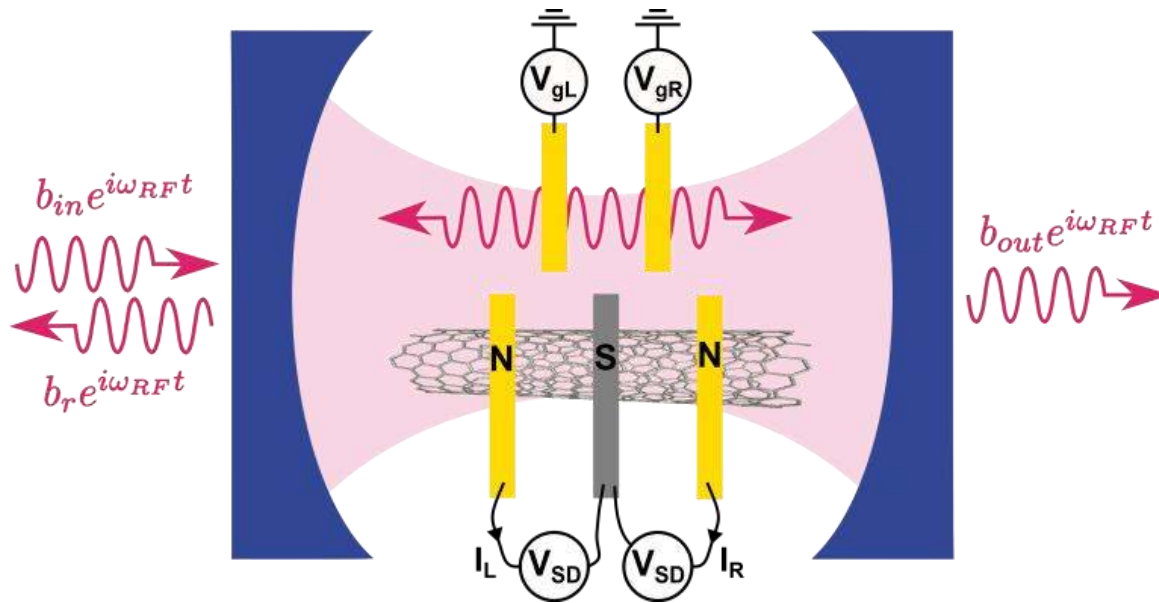
SCALING BEHAVIOR



Good theoretical modelling : first quantitative experimental test of cavity signals modelling in terms of **charge susceptibility**

→ opens an avenue because very general method !

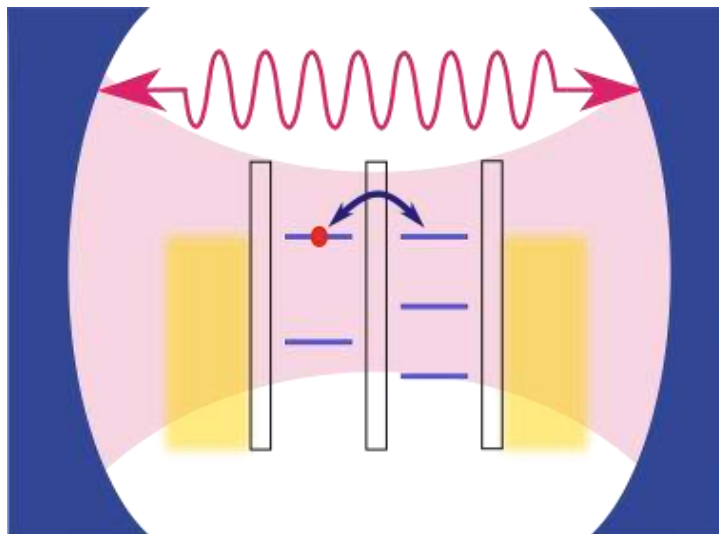
We developped the tools to put MORE PIECES in the big puzzle of science!



NEW!

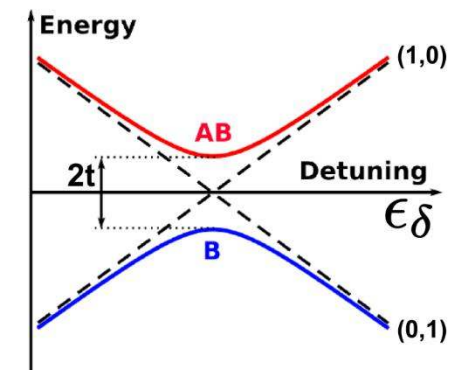
First time such a circuit was placed in a cavity!

Atomic-like device probed by a cavity

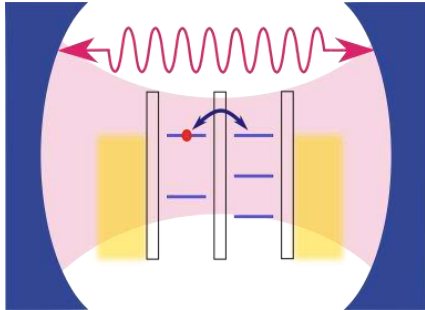


THE CLASSIC!

Most reproduced experiment in mesoscopic QED!

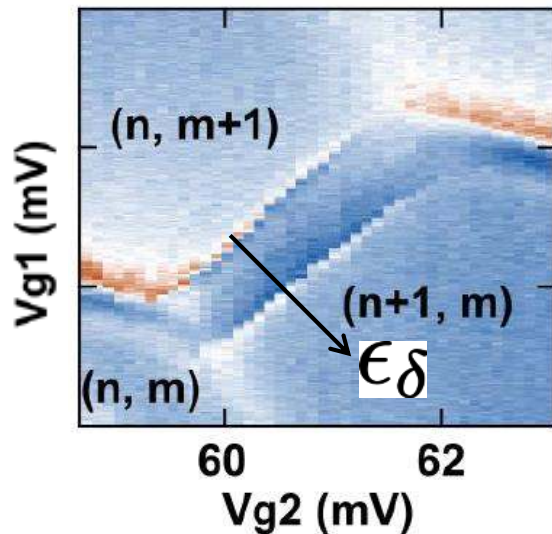


The double dot charge qubit



THE CLASSIC!

Phase (deg)
-10 0



Sign change in phase

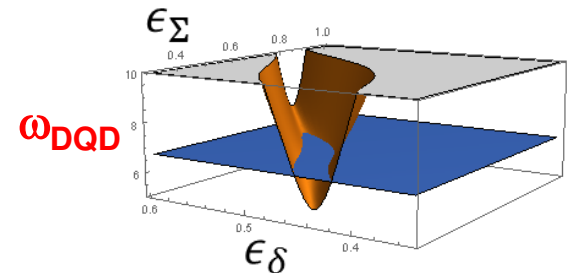
=

Resonant condition

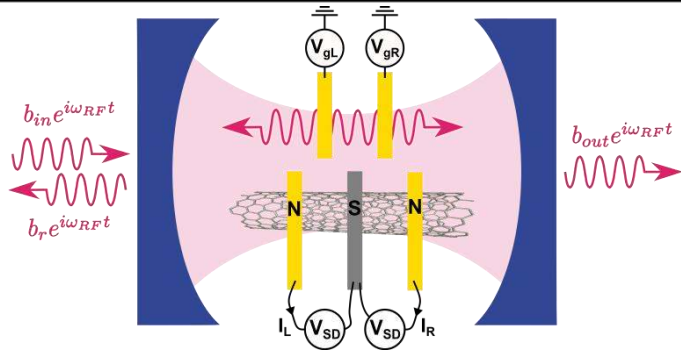
$$hf = \hbar\Omega$$

Double dot charge qubit:

$$\Omega(\epsilon_\delta)$$

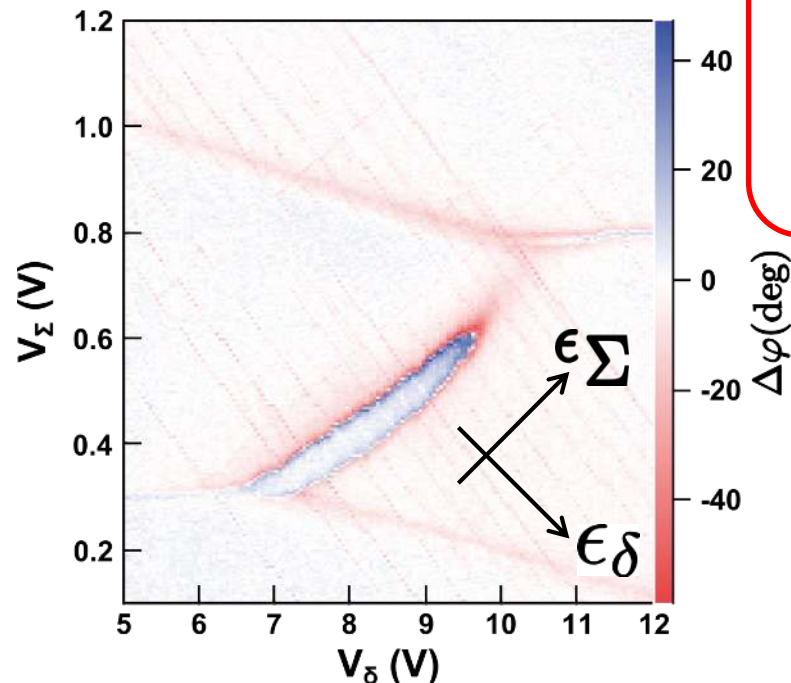


Use the cavity as a spectroscopic probe



NEW!

First time such a shape is observed!

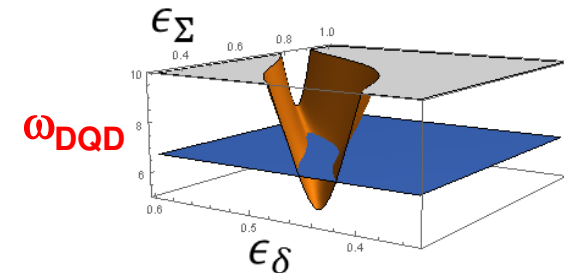


Sign change in phase

=

Resonant condition

$$hf = \hbar\Omega$$



Superconductor-double quantum dot:

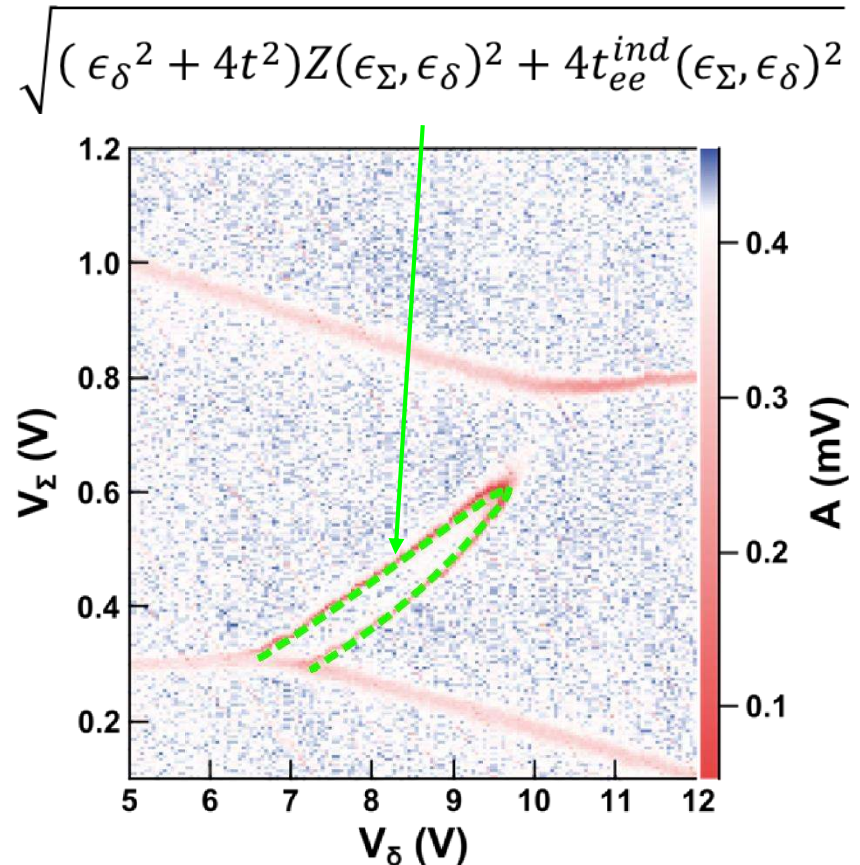
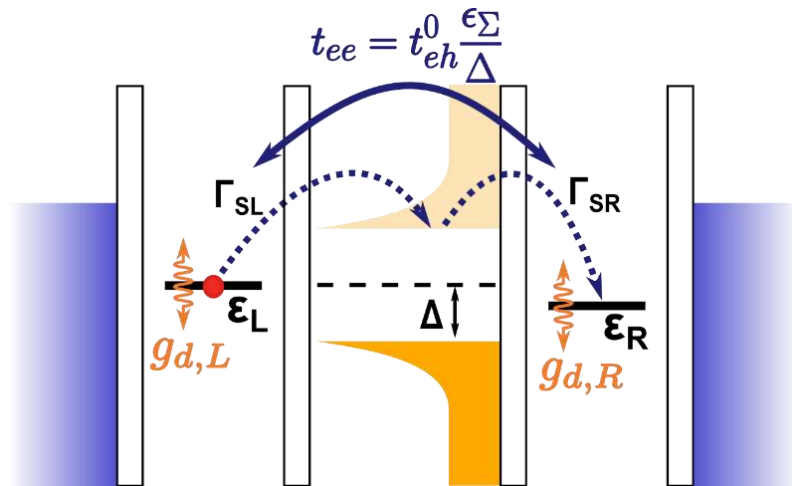
$$\Omega(\epsilon_\delta, \epsilon_\Sigma)$$

Sign of the modification of the spectrum by the superconductor ?

$$\Gamma_{SL}=800\text{MHz}, \Gamma_{SR}=400\text{MHz}$$

$$2t=6.3 \text{ GHz}, t_{eh}^0=400\text{MHz}$$

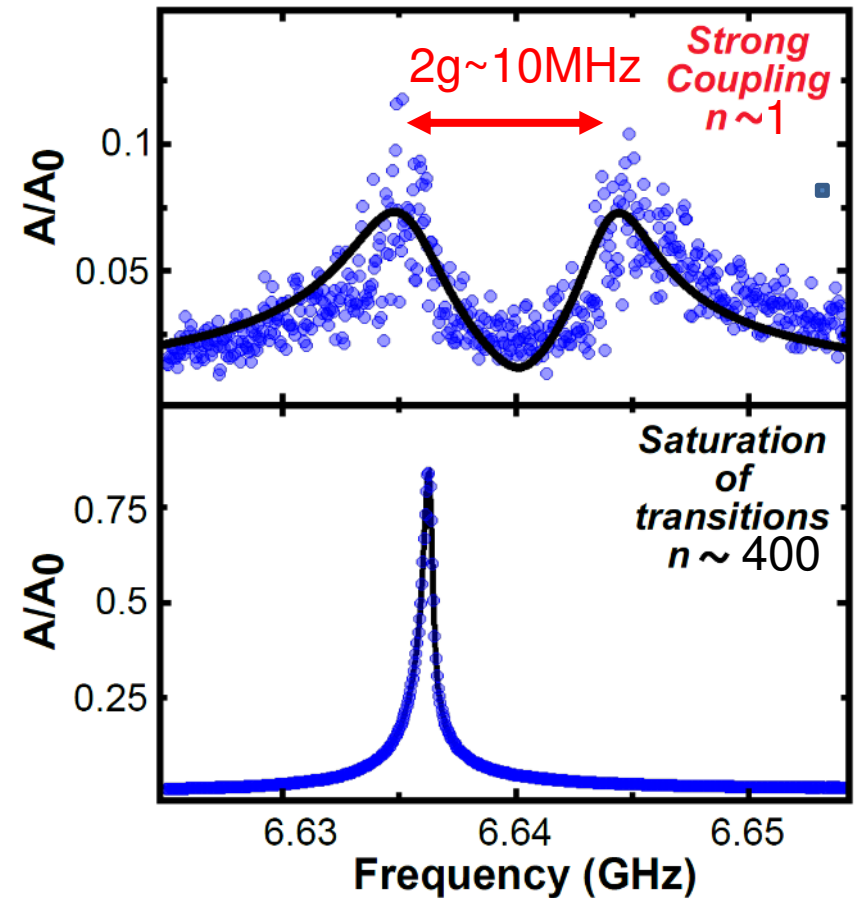
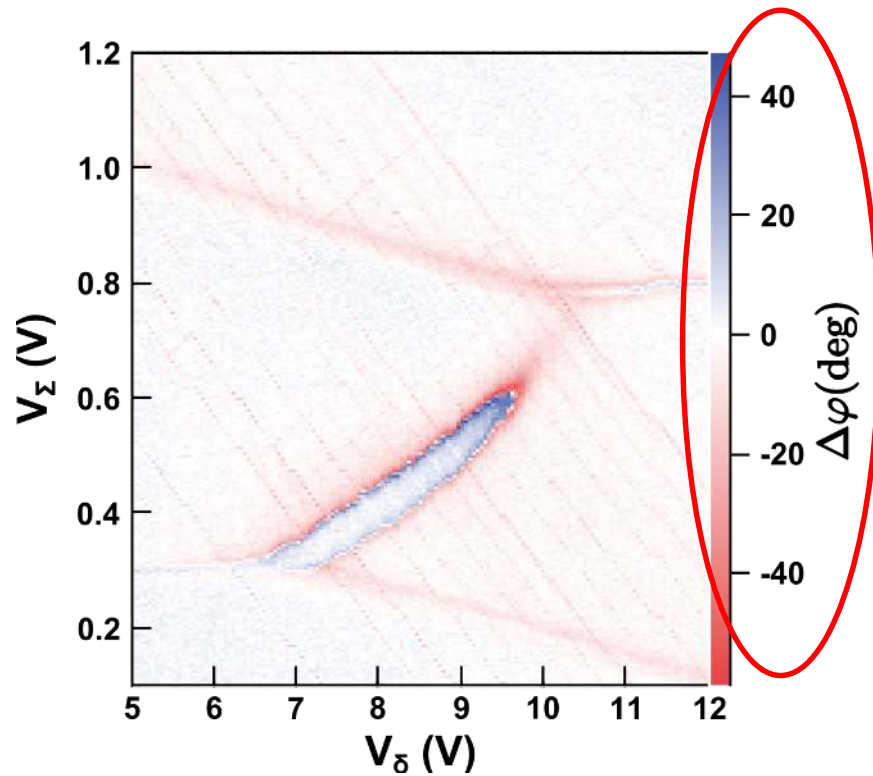
Role of Cooper pair assisted interdot tunneling



➡ Quantitative agreement with S-induced low energy spectrum.

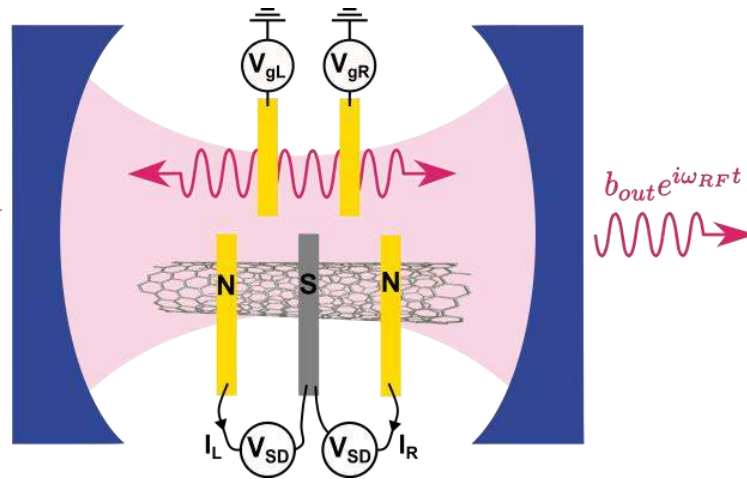
Observed resonance interpreted as the **transition between (0,1) and (1,0) states** « dressed » by the superconductor (odd eigenstates).

Last big surprise: Vacuum Rabi splitting!



Largest Vacuum Rabi splitting about 10 MHz ~ 3 line widths

Simultaneous observed by 3 groups



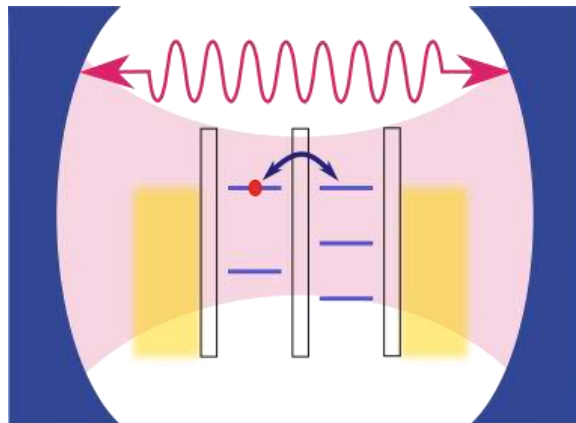
NEW!

- Observed with the first sample!
Bruhat et al., arXiv 2016
PRB 2018
- Built-in key features to reach the **strong coupling regime!**

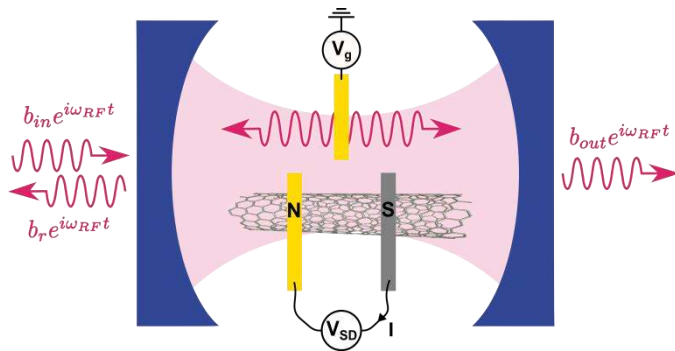
THE CLASSIC!

- Achieved in standard double quantum dots after years of effort.

Mi et al., Science 2017
Stockklauser et al., PRX 2017



I. Normal/dot/superconductor circuit in cavity

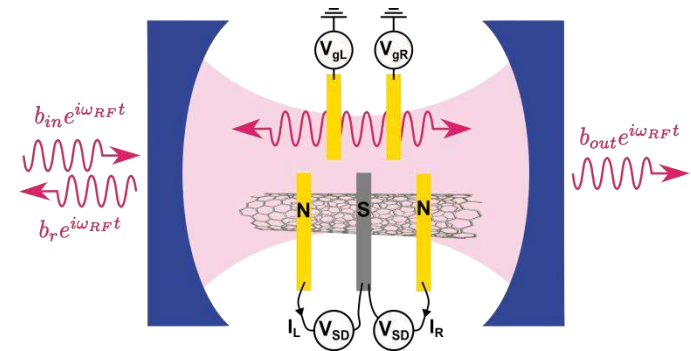


Open quantum dots :

Study quantum transport in a different way

- Kondo resonance *MM Desjardins et al Nature 2017*
- Majorana fermions

II. Superconductor-double quantum dot in cavity



Closed quantum dots:

Probe/manipulate internal quantum states of a mesoscopic circuit

- Applications in **quantum information processing**

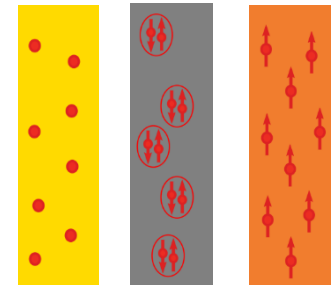
Rich possibilities offered by the combination of different metallic electrodes

see review by **A. Cottet et al, *J. Phys. Condens. Matter* 29, (2017).**

Normal (N)

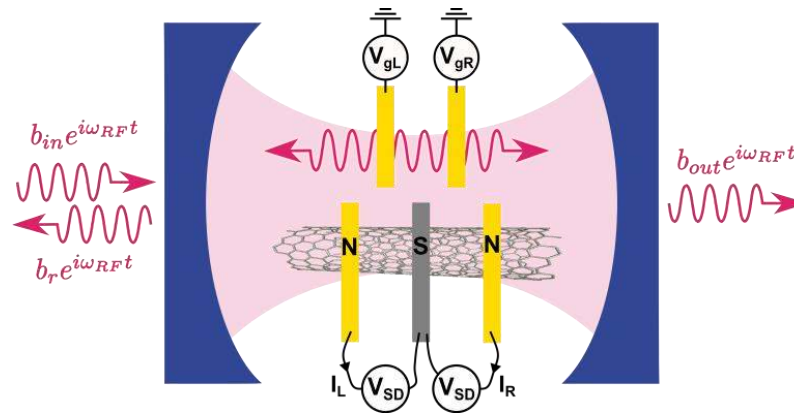
Superconducting (S)

Ferromagnetic (F)



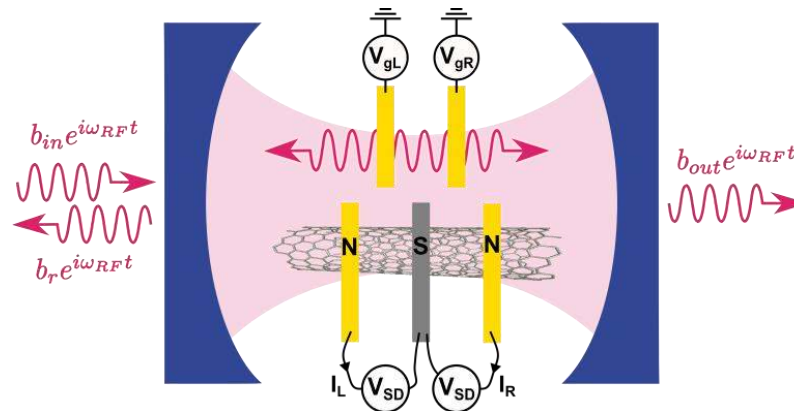
Was this all planned?

INPUT: YES!



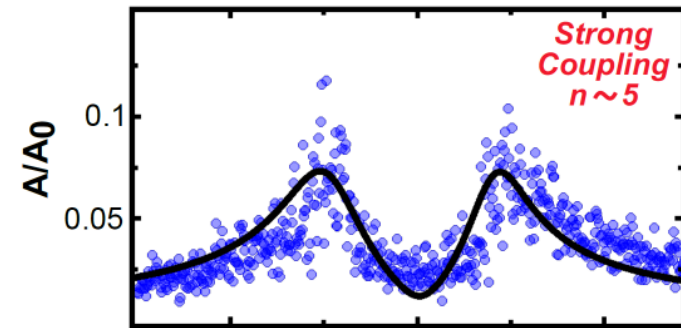
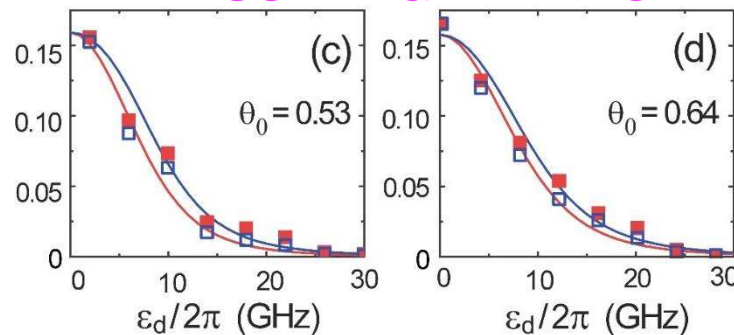
Was this all planned?

INPUT: YES!



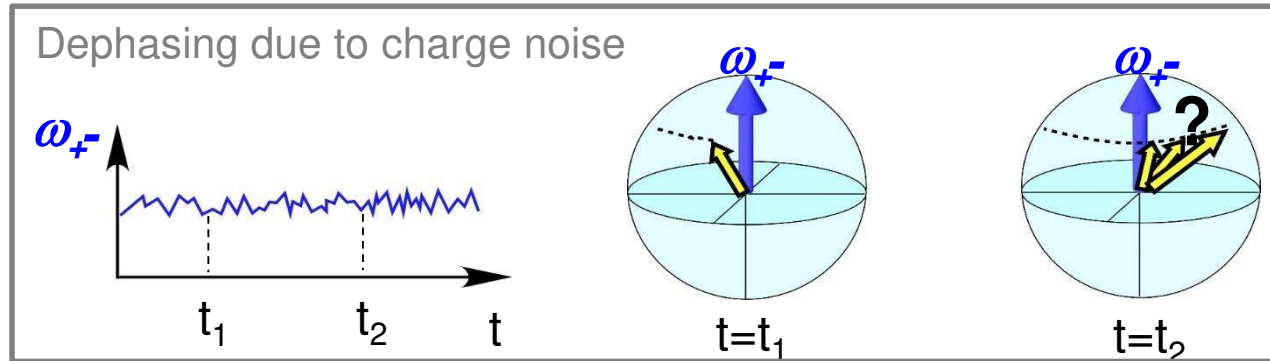
OUTPUT: NO!

SCALING BEHAVIOR



I wish you many good surprises in your research!
Be receptive!

Strategies to reach the strong coupling regime



$$g_{\perp} \ll \Gamma_s$$

How do we get $g_{\perp} \gg \Gamma_s$?

Increase $g_{\perp}$

Decrease Γ_s

$$\Gamma_s \sim \frac{1}{\pi} \frac{\partial^2 \omega_{+-}}{\partial \varepsilon^2} E_c^2 (\delta n)^2$$

decrease E_c

Bruhat et al., arXiv 2016

Paris, carbon nanotube

$$\tilde{g}_{\perp} / \Gamma_s = 2.5$$

decrease charge noise

Mi et al., Science 2017

Princeton, Si 2DEG

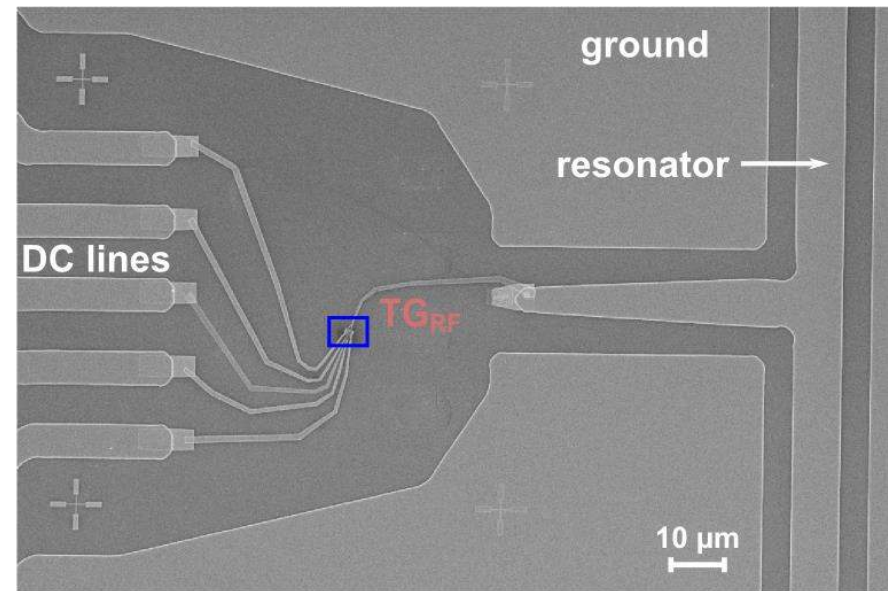
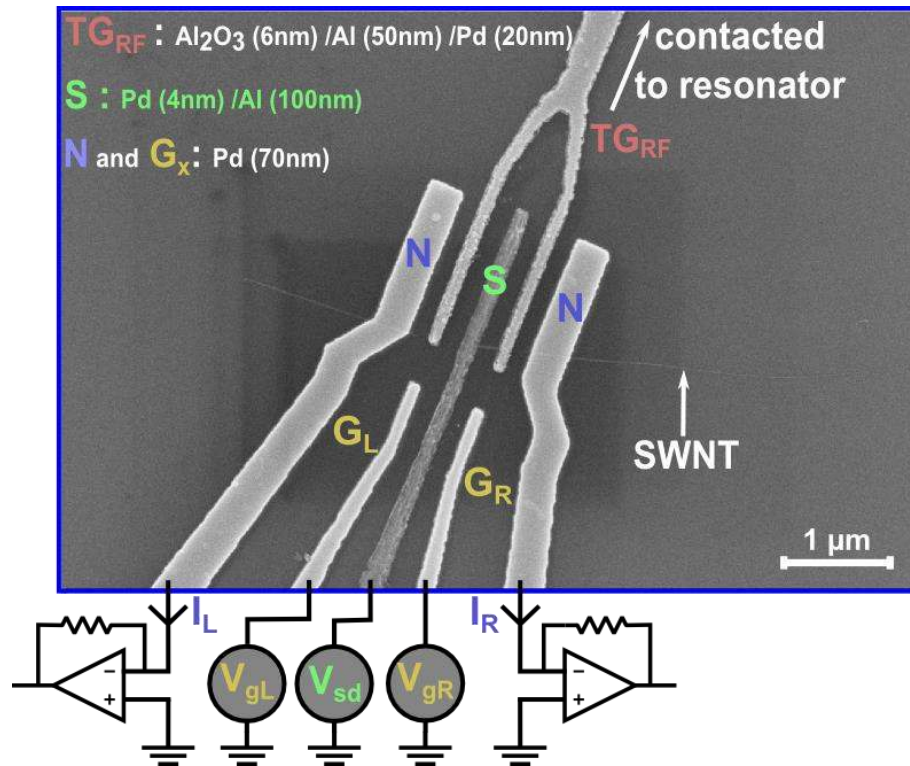
$$g_{\perp} / \Gamma_s = 2.57$$

Stockklauser et al., PRX 2017

Zurich, GaAs 2DEG

$$g_{\perp} / \Gamma_s = 1.27$$

Key ingredient 1: decrease E_c



Key features

- Double dot with a central superconducting contact
- **Fork top gate** to resonator

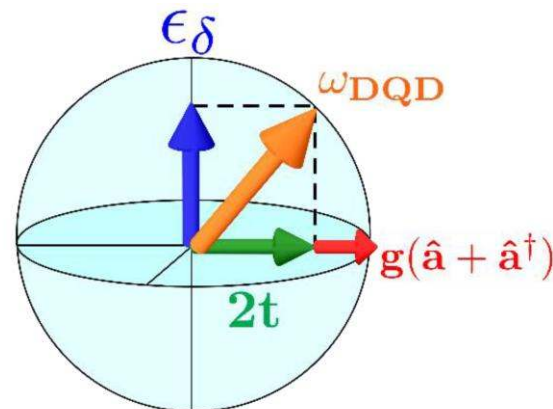
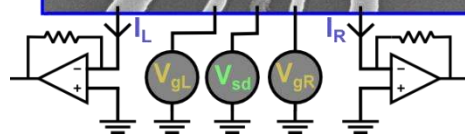
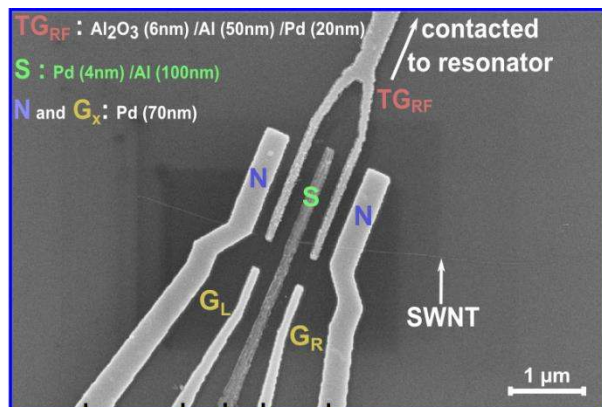
$$E_c = \frac{e^2}{2C_{dot}}$$

For $E_c \approx 10$ meV, $\Gamma_2 \approx 550$ MHz

See JJ Viennot et al, PRB 89, 165404 (2014)

Here $E_c \approx 1$ meV, $\Gamma_2 \approx 5$ MHz

Key ingredient 2: coupling via interdot hopping



Key features

- Double dot with a central **superconducting contact**
- **Fork** top gate to resonator

$$\tilde{t} = t + \underbrace{t_{ee}^{ind}}_{\text{intrinsic} \propto \frac{\epsilon \Sigma}{\Delta}}$$

Renormalization of L/R hopping from elastic cotunnelling through superconductor

$$\epsilon \Sigma = \epsilon_L + \epsilon_R \propto \alpha V_{gL} + \beta V_{gR} + V_{ac}(a + a^\dagger)$$

large transverse coupling via **symmetric coupling to em field**

