# Local mechanical description of a single elastic fold

PHAST
PHYSIQUE
ET ASTROPHYSIQUE
UNIVERSITÉ DE LYON
ÉCOLE DOCTORALE 52

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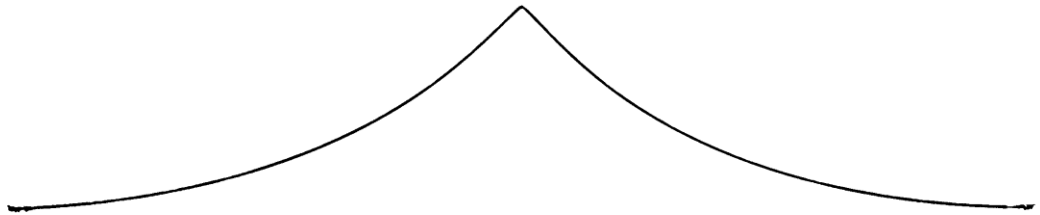
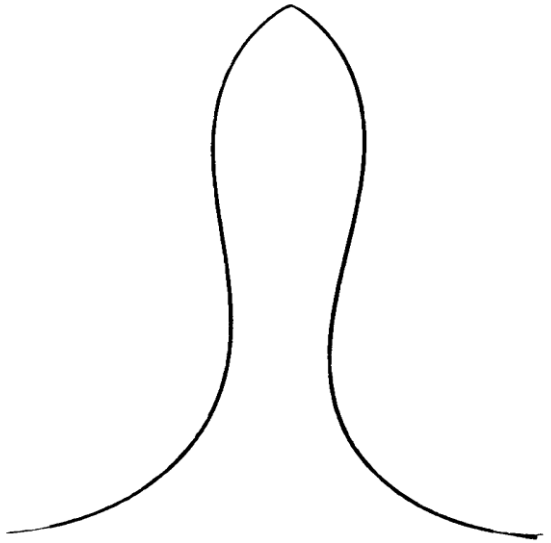


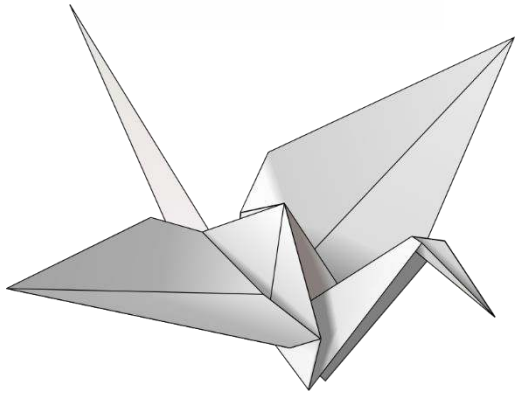
Local mechanical description of an Elastic Fold
*Jules et al, arXiv:1808.04892*



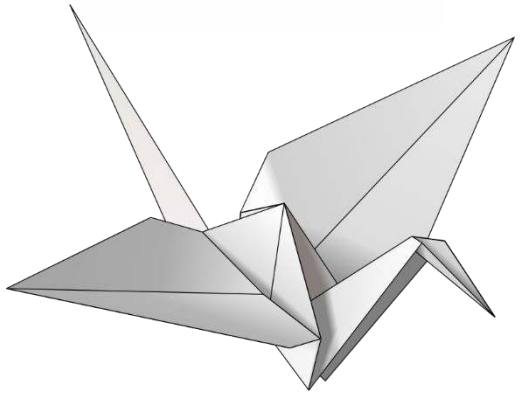
Département de Physique
École normale supérieure

Lab PHYS ENS de LYON

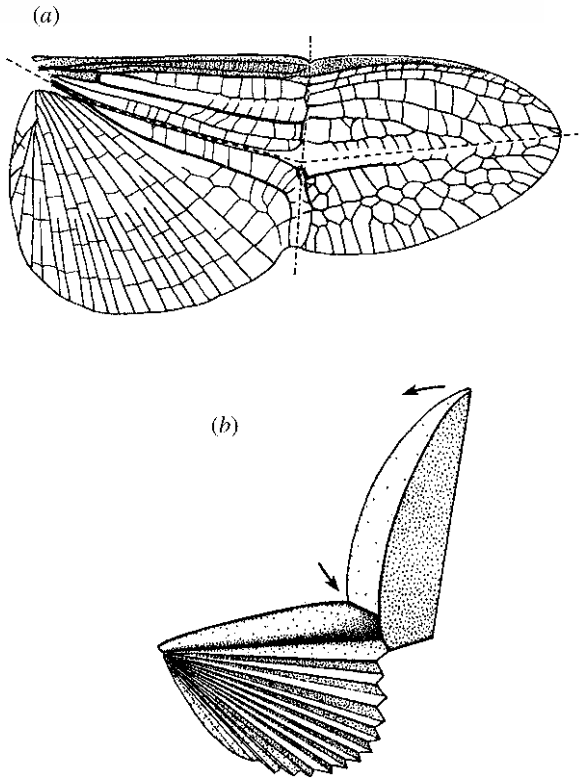
# Art



# Art



# Nature




*Haas and Wootton, PRSLB 1996*

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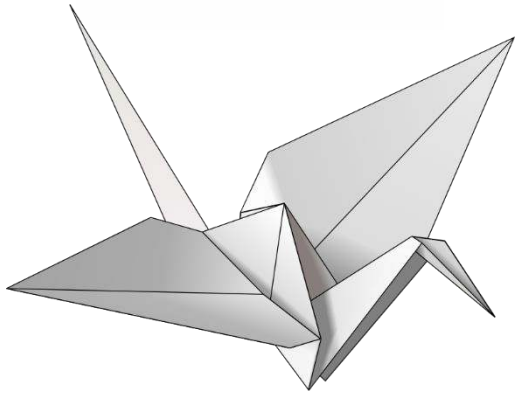

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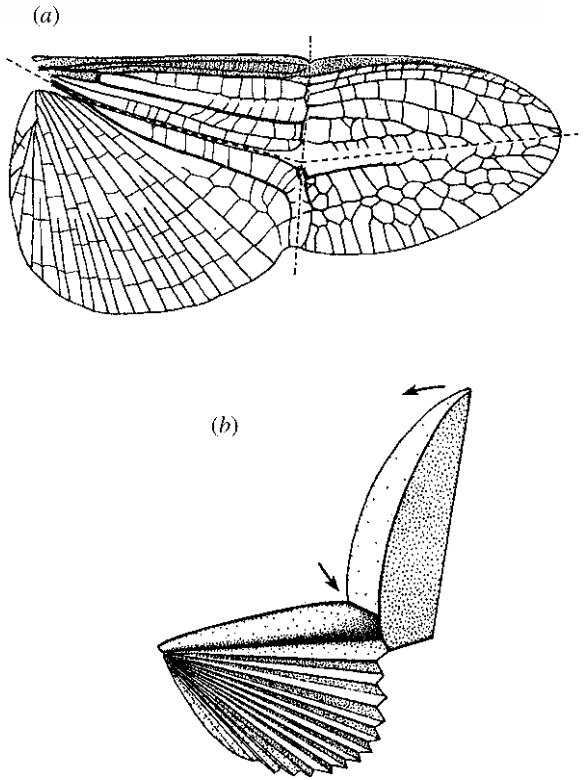
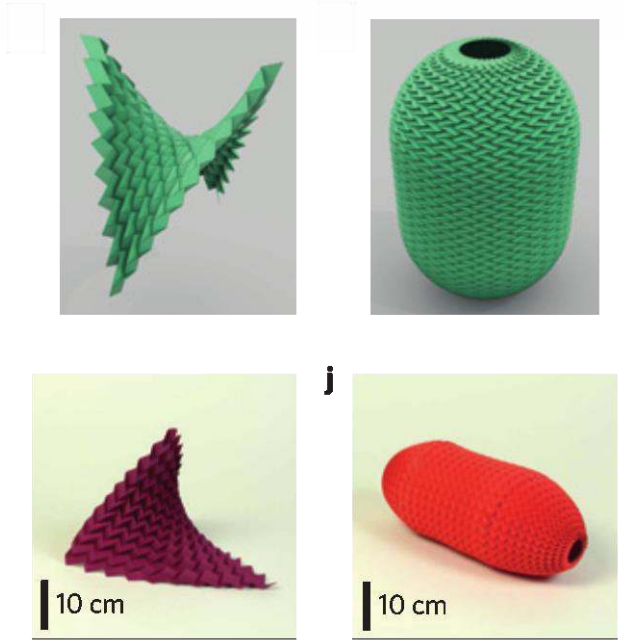


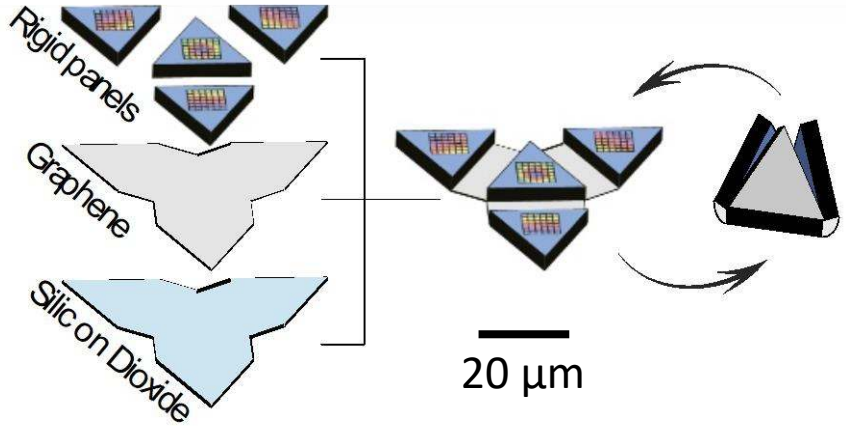

*Haas and Wootton, PRSLB 1996*

# Engineering



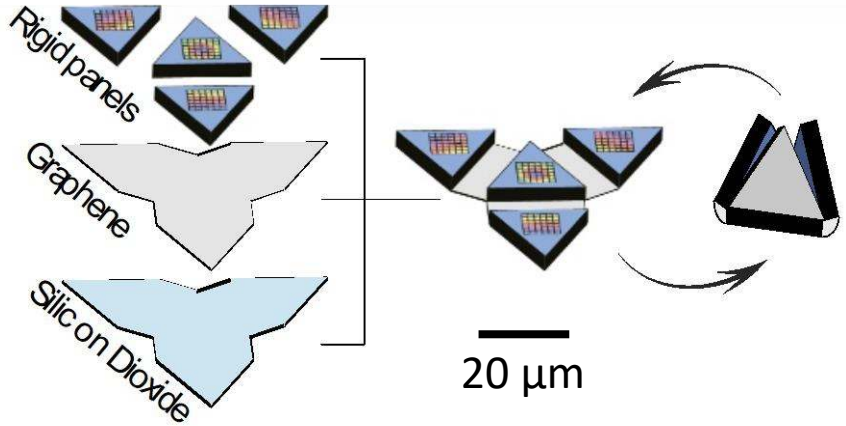

*Dudte et al., Nature material 2016*

# Microscale




Micron-sized autonomous origami machine,
*Minsink et al. PNAS 2018*

# Microscale

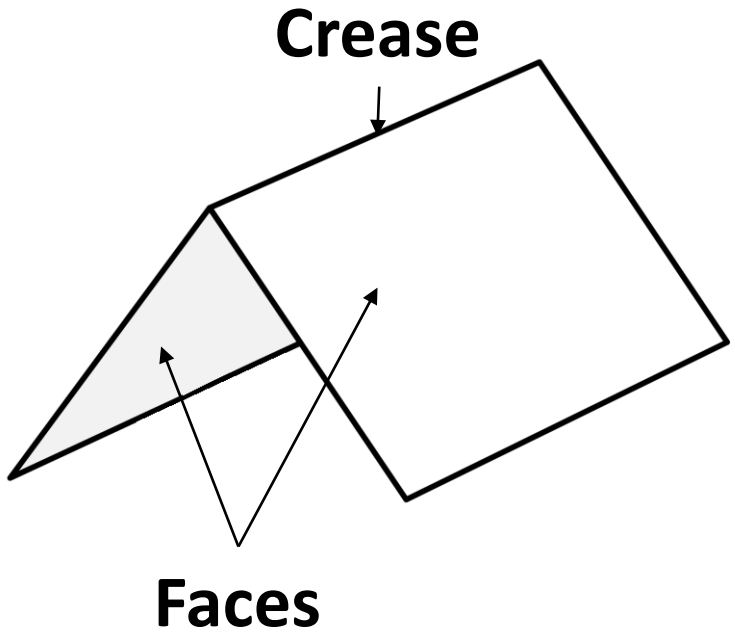



Micron-sized autonomous origami machine,
*Minsink et al. PNAS 2018*

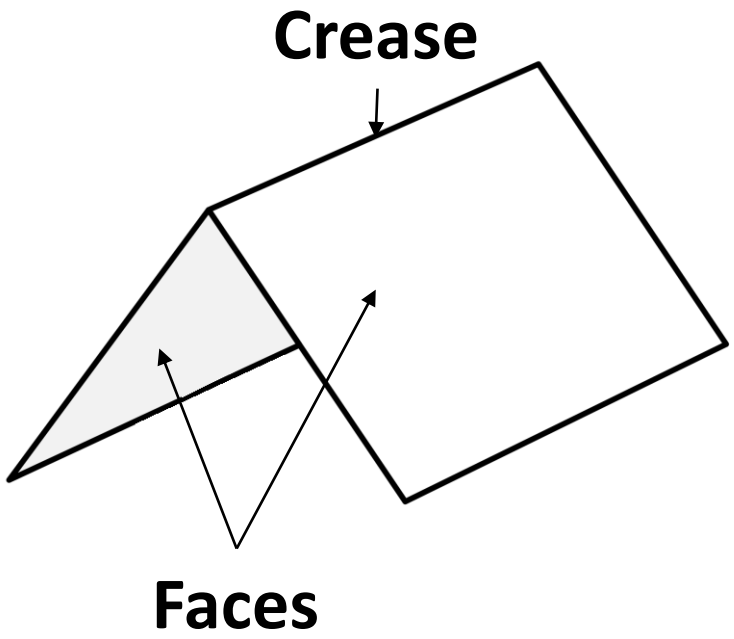
# Macroscale


Deployable starlight shield,
*NASA, 2018*

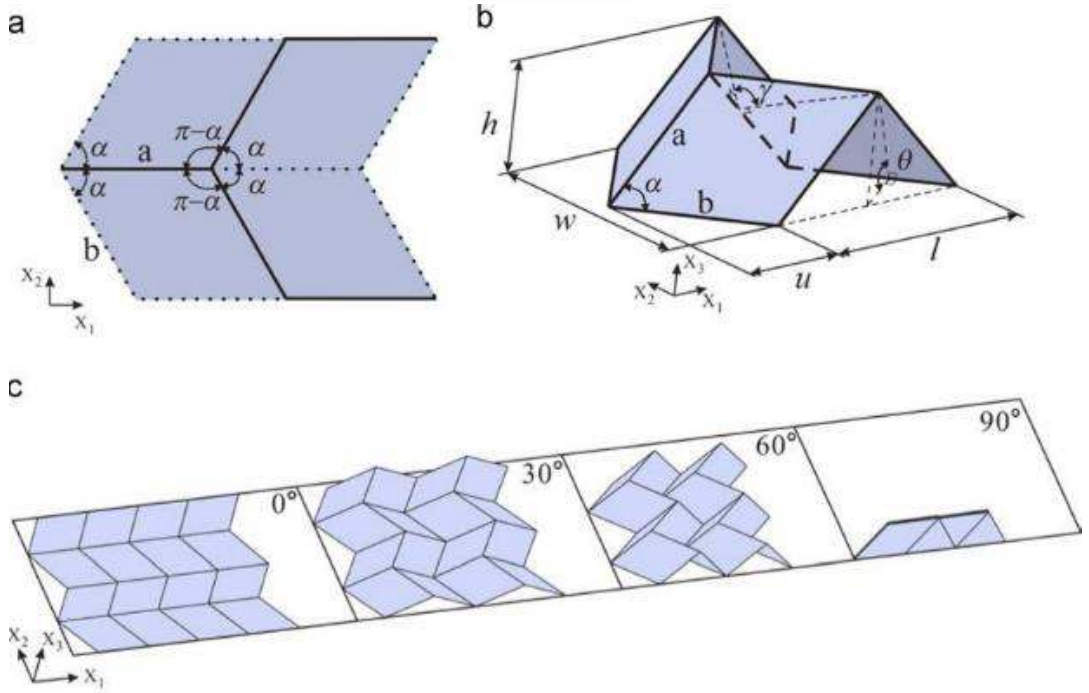
# Single fold



# Single fold




# Multiple creases




Miura-Ori cell,
*Liu et al, IJMS 2015*

# Before actuation

Miura-Ori pattern

# Before actuation

Miura-Ori pattern

# After actuation

Miura-Ori pattern

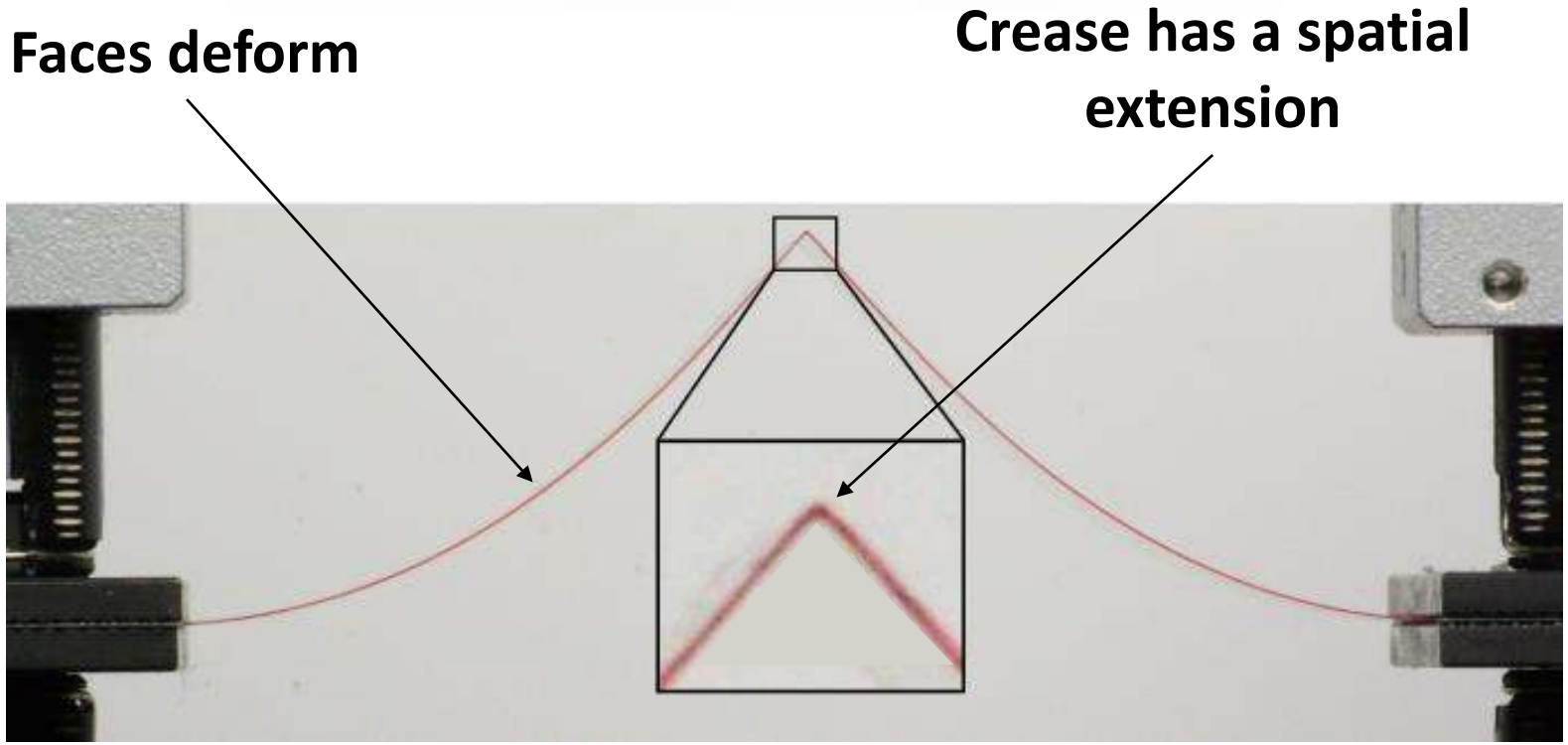
# Before actuation

# After actuation

Miura-Ori pattern


Miura-Ori pattern

# Single elastic fold

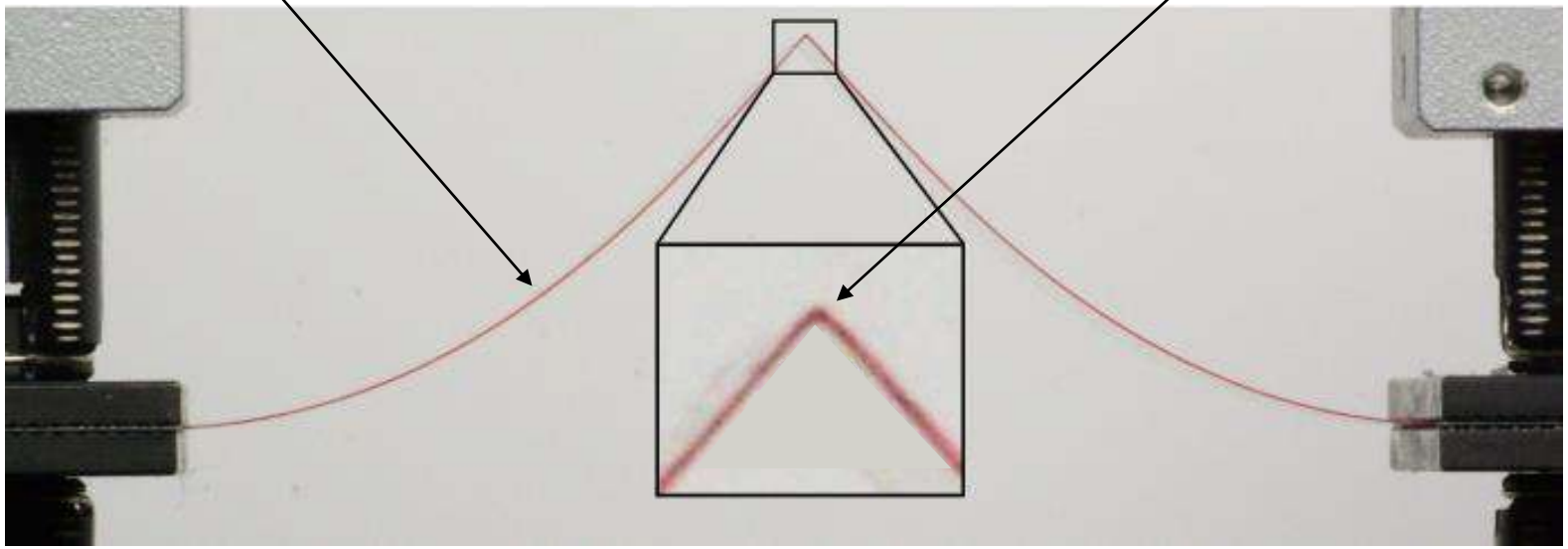



***Experiment** : Single fold under external load*

# Single elastic fold

**Faces deforms**

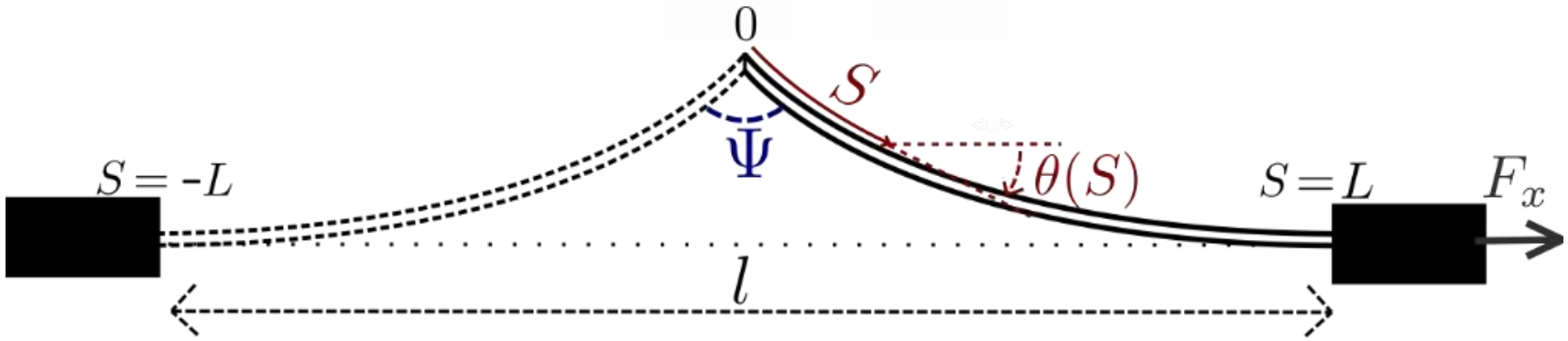
**Crease has a spatial extension**



***Experiment** : Single fold under external load*

- Shape of the deformed fold ?
- Define a crease rigidity ?

# Continuous fold model

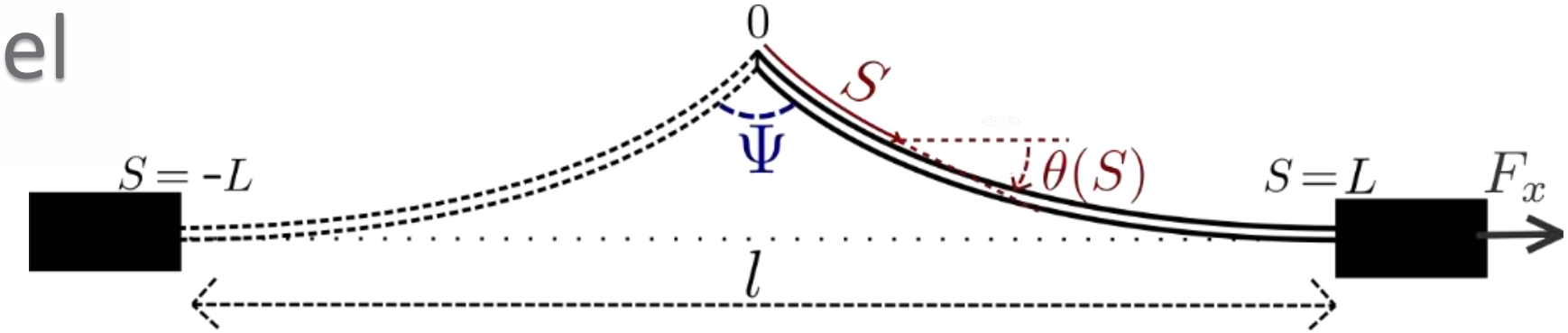



S: Curvilinear coordinates.

Θ(S): Angle between local tangent and horizontal line

Ψ: Macroscopic crease angle

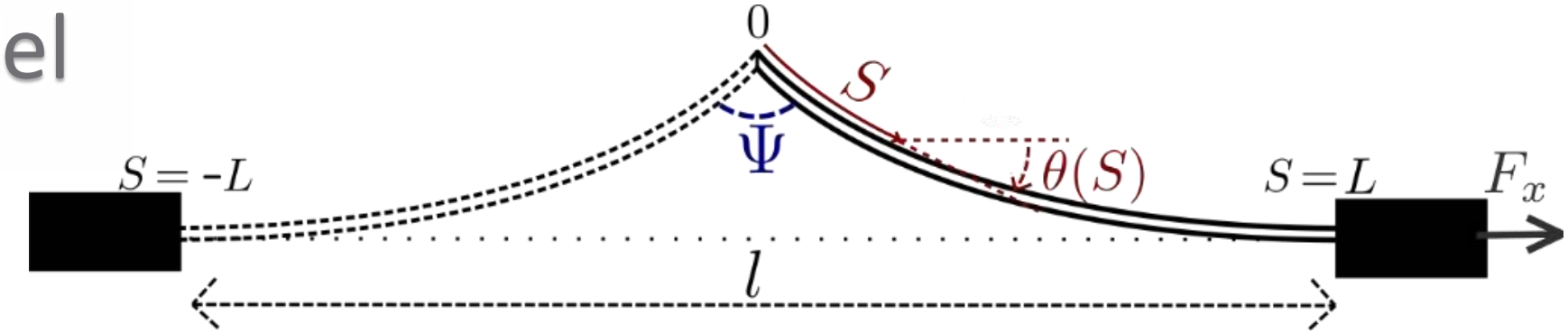
# Fold model




## Plastic crease = Pre-strain

$$E_{Bending} = \underbrace{\frac{BW}{2}}_{\text{Bending Rigidity}} \left( \underbrace{\theta'(S)}_{\text{Local Curvature}} - \underbrace{\theta_0'(S)}_{\text{Reference Curvature}} \right)^2$$

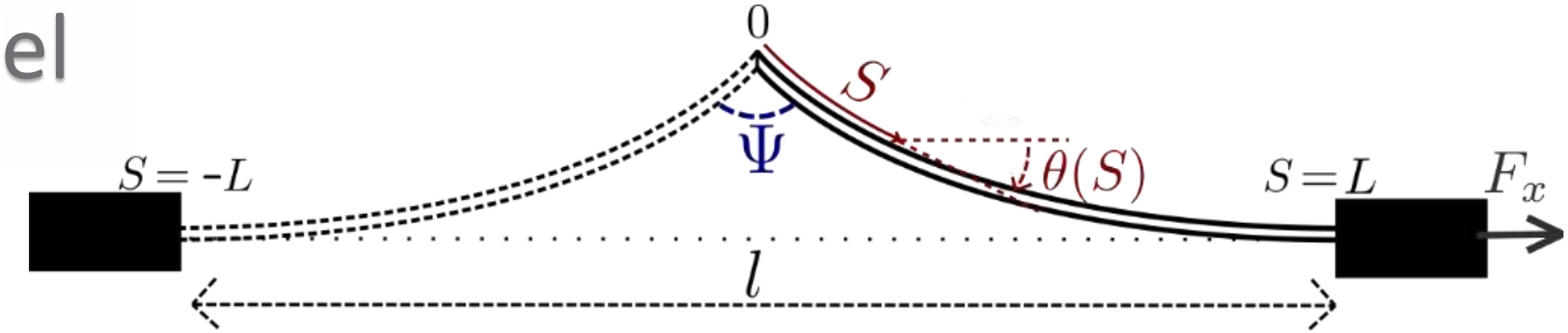
# Fold model




# Normalized Lagrangian

$$\mathcal{L} = \underbrace{\frac{BW}{2L}\int_{-1}^{1}(\theta'(s)-\theta_0'(s))^2\,ds}_{\text{Elastic Energy}} \; \underbrace{- F_x L\int_{-1}^{1}\cos(\theta(s))\,ds}_{\text{External Work}}$$

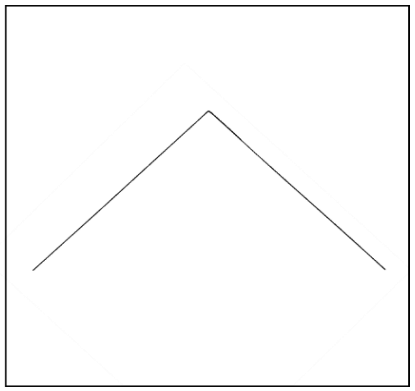
# Fold model




# Shape: pre-strained Elastica

$$\theta'' - \theta_0'' - \frac{F_x L^2}{BW}\sin\theta = 0$$

# Reference Shape



Fold is put at rest

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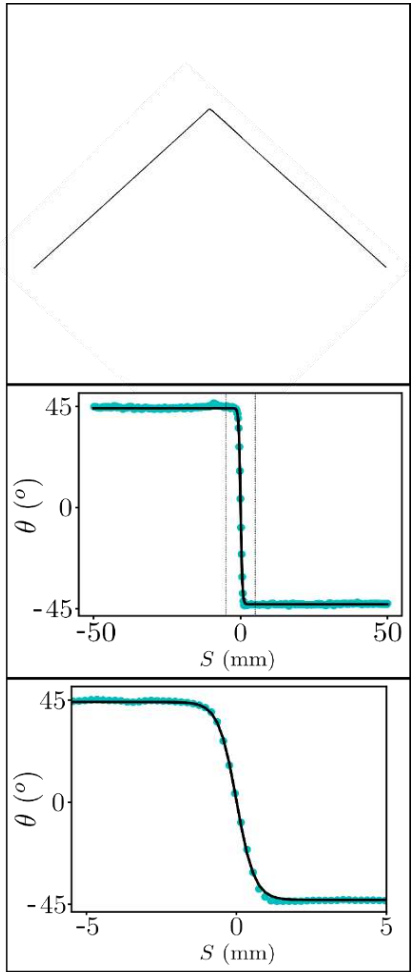
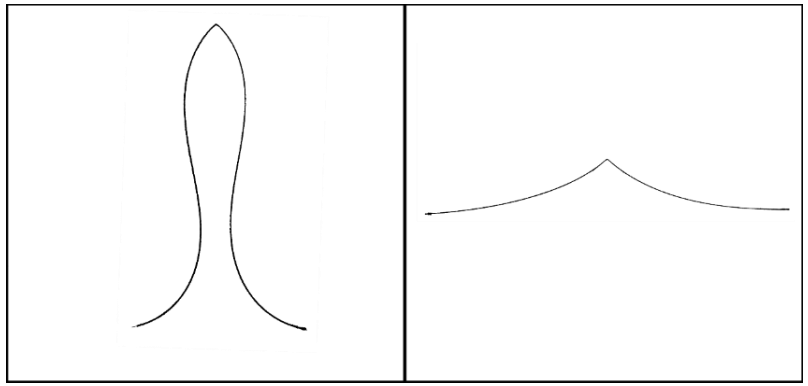



Fold is put at rest

## Fit by a continuous fonction:

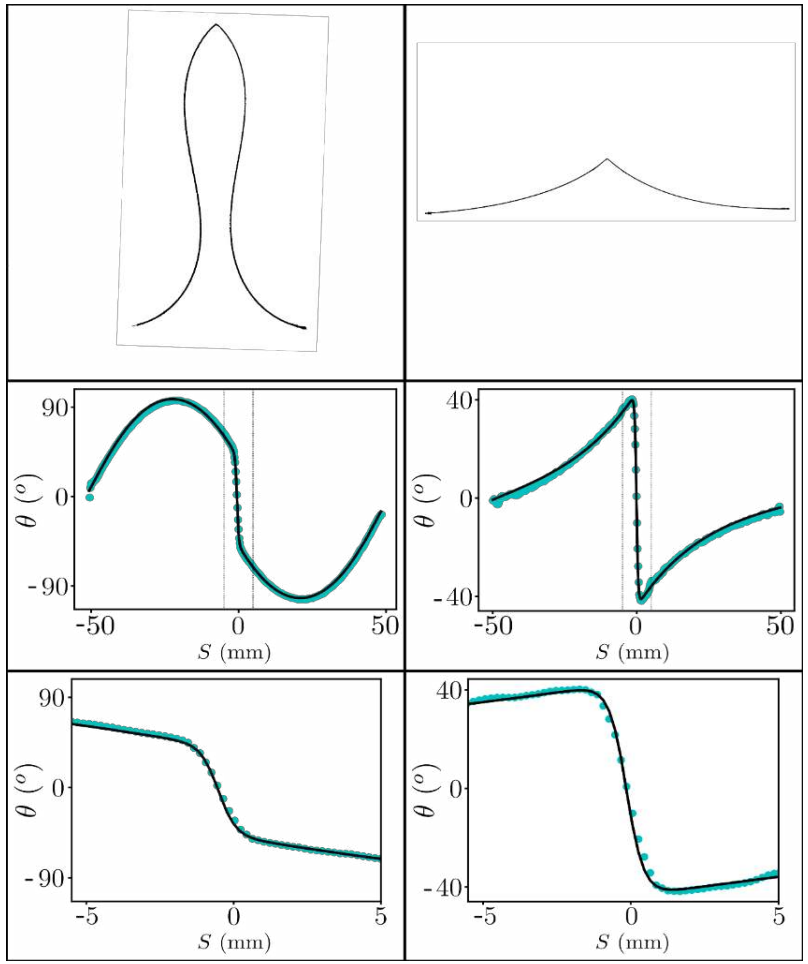
$$\theta_0(S) = \frac{\Psi_0 - \pi}{2}\left[\tanh\left(\frac{S}{S_0}\right)\right]$$



# Deformed Shape

Experimentally we reach both compressive (left) and tensile (right) regime.

# Deformed Shape




Numerical shape

Experimental shape

Experimentally we reach both compressive (left) and tensile (right) regime.

Good agreement in both cases between numerical solutions and experimental shapes.

# Asymptotic Analysis

$$\theta'' - \theta_0'' - \frac{F_x L^2}{BW} \sin\theta = 0$$

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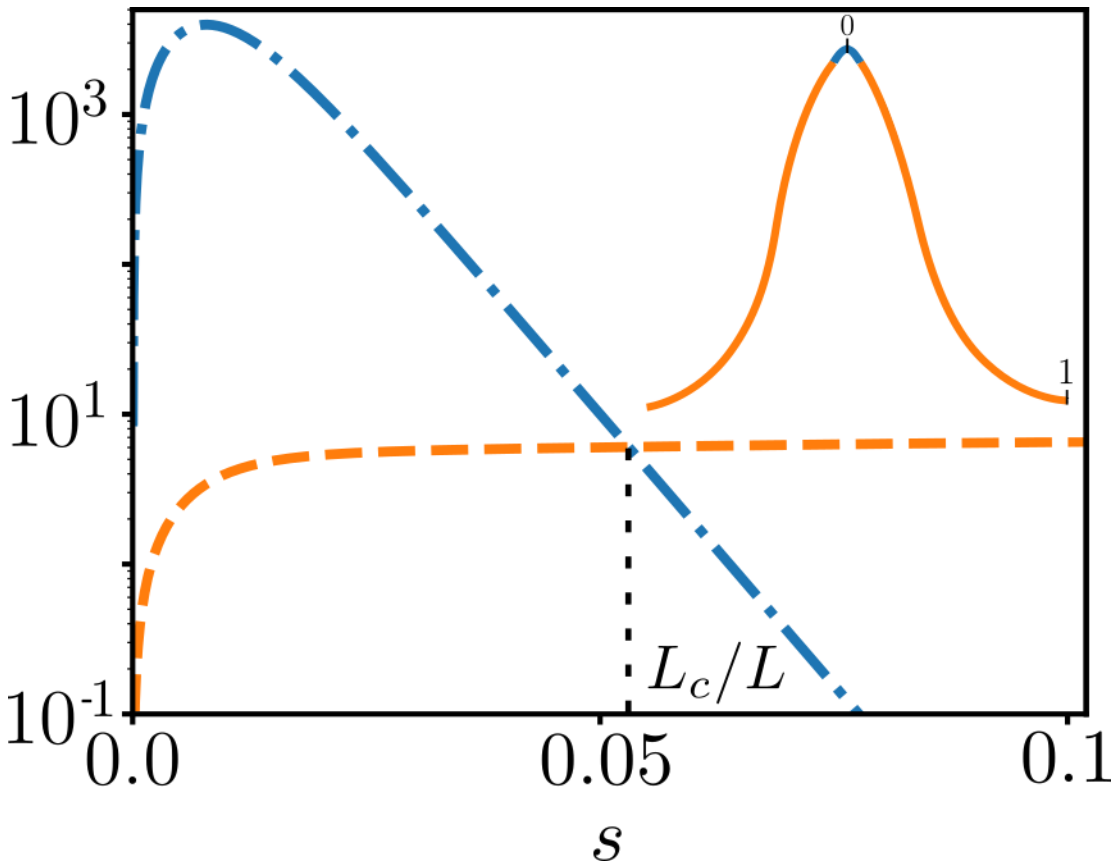
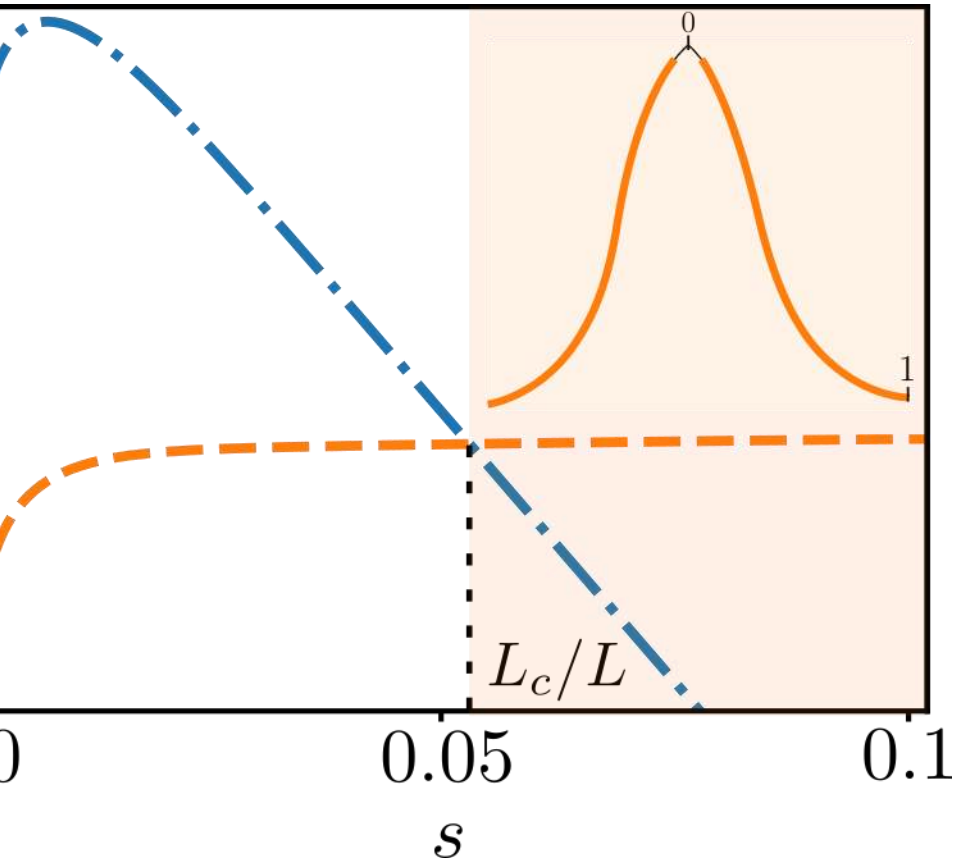



# Asymptotic Analysis

$$\theta'' - \theta_0'' - \frac{F_x L^2}{BW}\sin\theta = 0$$

**Case 1:** $\theta_0'' \ll \frac{F_x L^2}{BW}$

$$\theta'' - \frac{F_x L^2}{BW}\sin\theta = 0$$

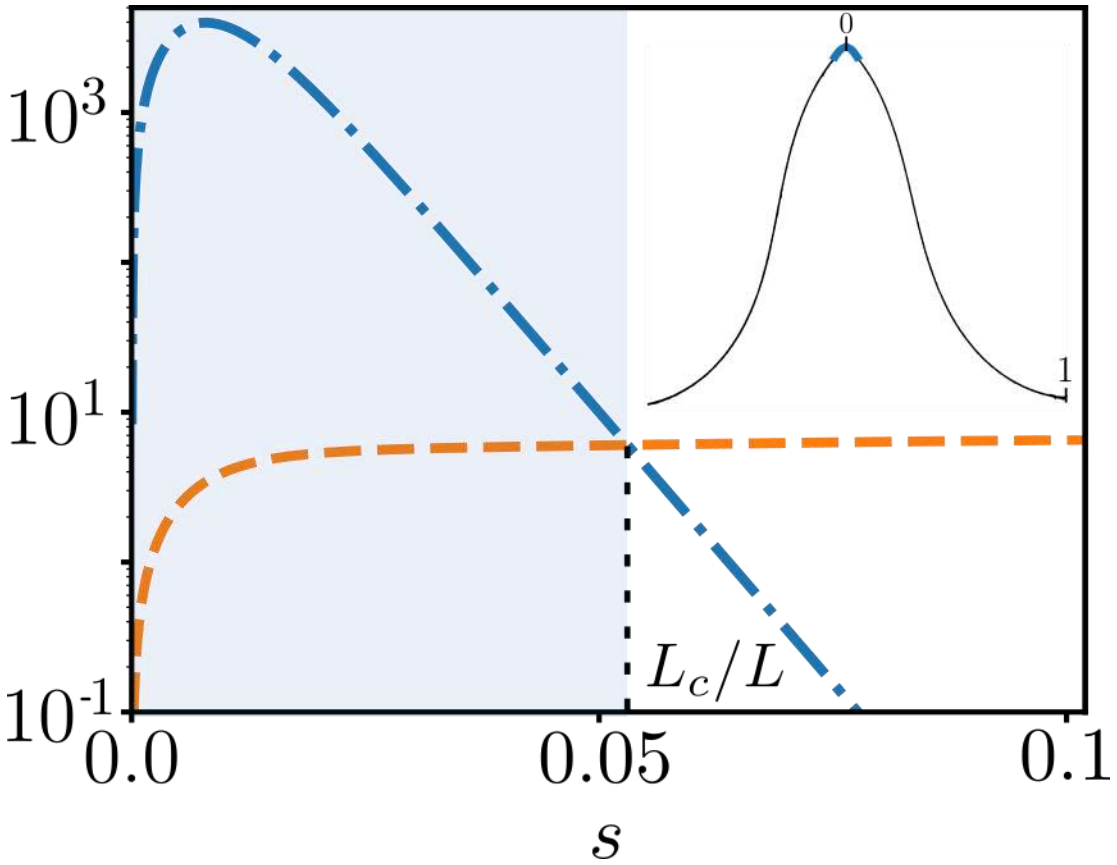


# Asymptotic Analysis

$$\theta'' - \theta_0'' - \frac{F_x L^2}{BW}\sin\theta = 0$$

**Case 2:** $\theta_0'' \gg \frac{F_x L^2}{BW}$

$$\theta'' - \theta_0'' = 0$$




# Asymptotic Analysis

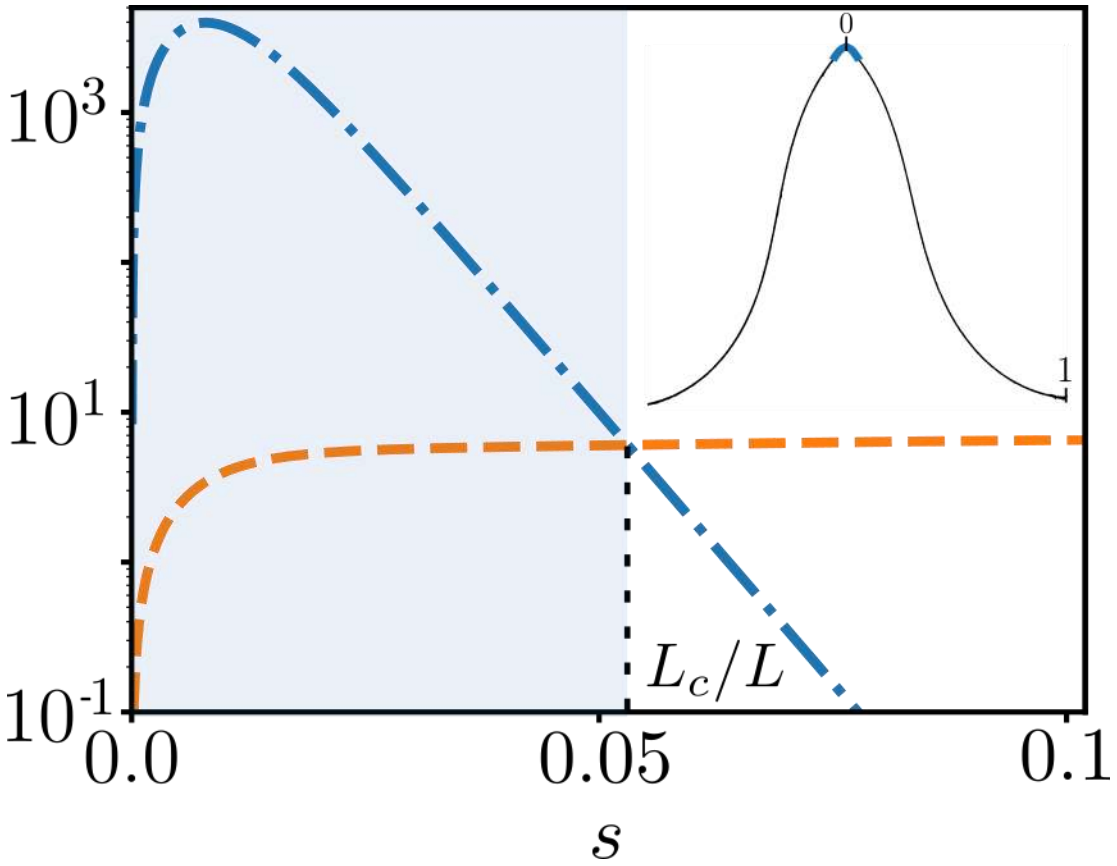
$$\theta'' - \theta_0'' - \frac{F_x L^2}{BW}\sin\theta = 0$$

**Case 2:** $\theta_0'' \gg \frac{F_x L^2}{BW}$

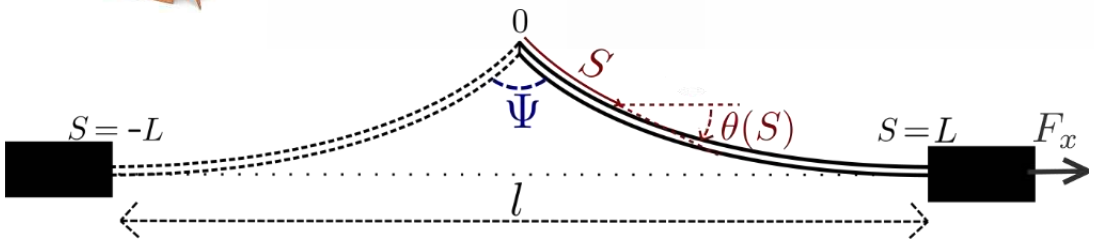
$$\theta'' - \theta_0'' = 0$$

$$\Downarrow$$

$$(\theta - \theta_0)'(L_c) = \frac{1}{L_c}(\theta - \theta_0)(L_c)$$



# From continuity to discontinuity



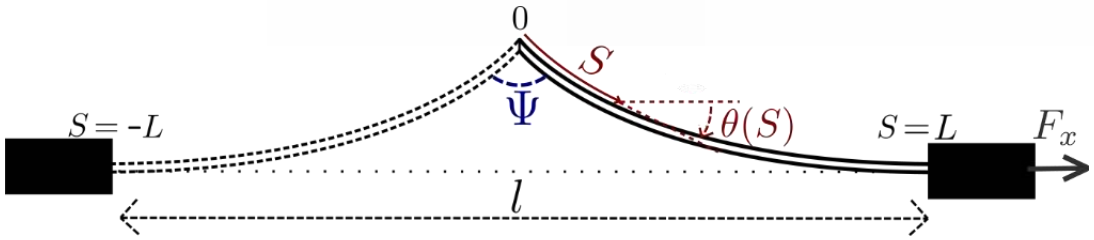

## End of the crease: S = Lc

$$\Psi = \pi + 2\theta(L_c)$$

$$M_{crease} = BW(\theta - \theta_0)'(L_c)$$

$M_{crease}$ : Moment generated by the crease

# From continuity to discontinuity




## At the crease:

$$(\theta - \theta_0)'(L_c) = \frac{L}{L_c}(\theta - \theta_0)(L_c)$$

## End of the crease: S = Lc

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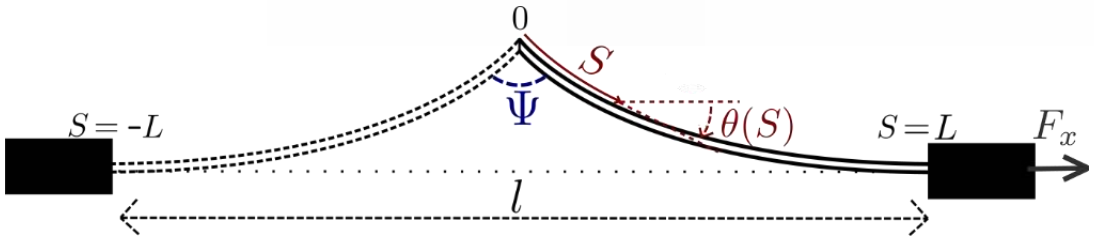


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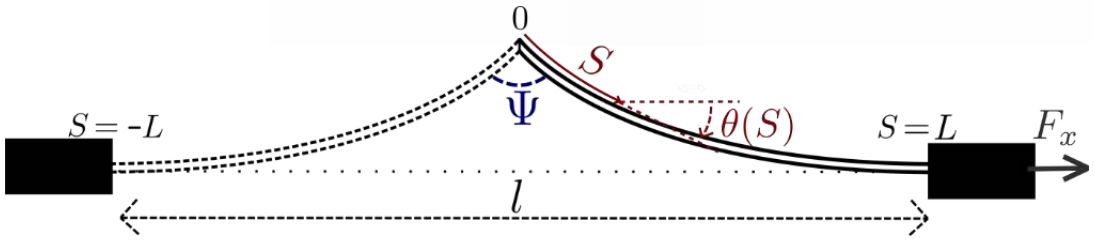



## At the crease:

$$(\theta - \theta_0)'(L_c) = \frac{1}{L_c}(\theta - \theta_0)(L_c)$$

## End of the crease: S = Lc

$$\Psi = \pi + 2\theta(L_c)$$

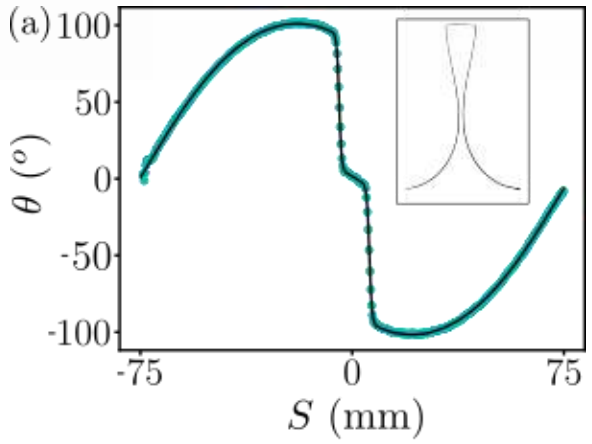
$$M_{crease} = BW(\theta - \theta_0)'(L_c)$$

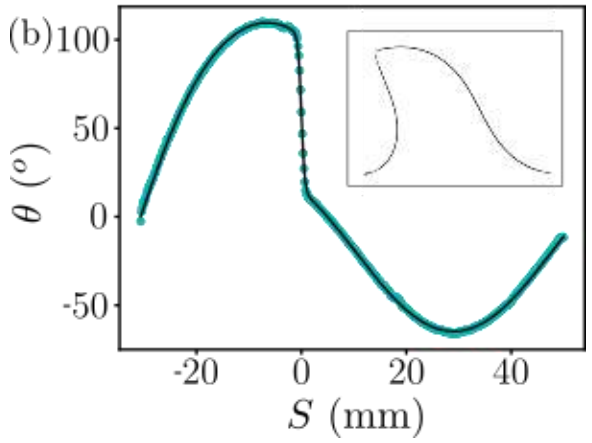
$M_{crease}$ : Moment generated by the crease

## Crease rigidity

$$M_{crease} = \boxed{\frac{BW}{L_c}}(\Psi - \Psi_0)$$

# Other applications of the model

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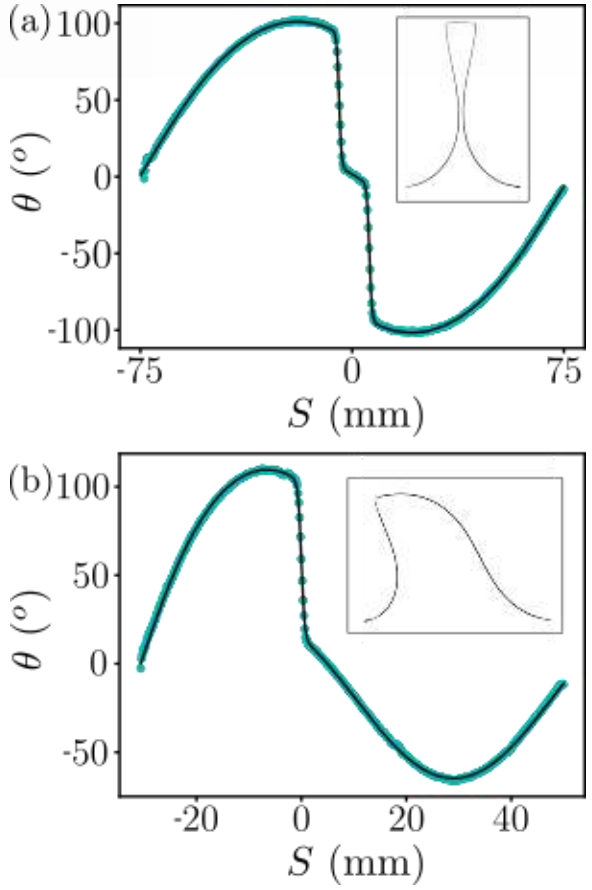
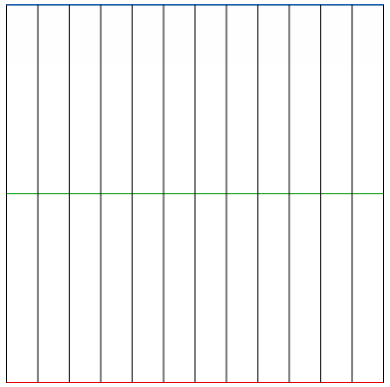
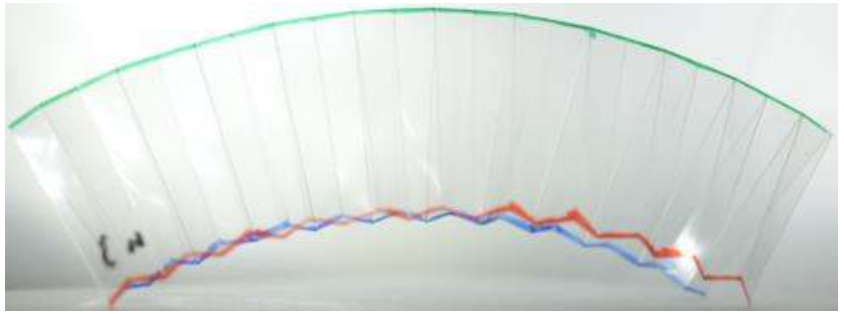



# Spontaneous curvature





# Thanks for your attention

***Origamis by Jo Nakashima***