# Playing with a photon

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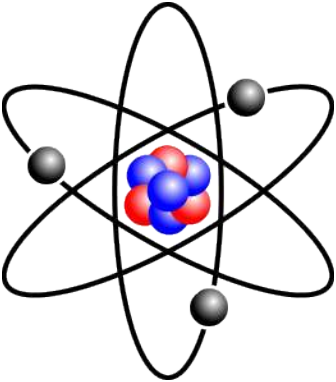
# Spoiler



Matter is made of atoms
(image : Wikipedia)

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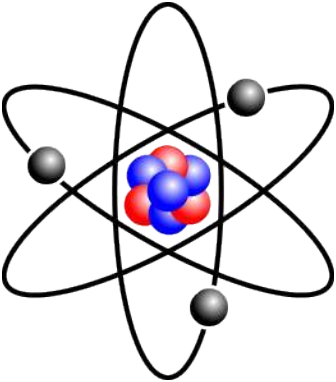

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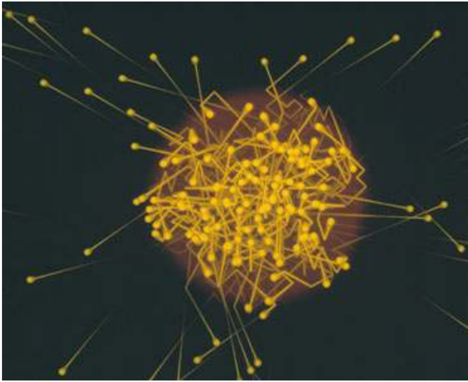


Light is made of photons
(image : CNRS)

# How to keep light for a while

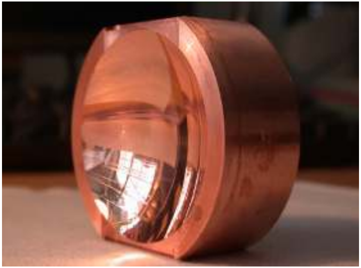
# Goal

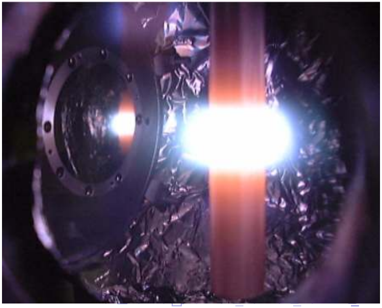
Prepare and probe a delocalized state of one photon stored in two cavities :

$$\frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right).$$

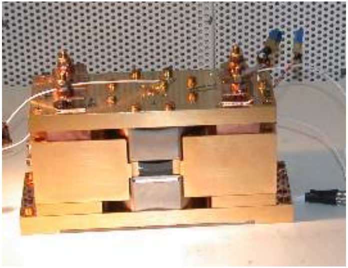
# Designing highly reflective mirrors

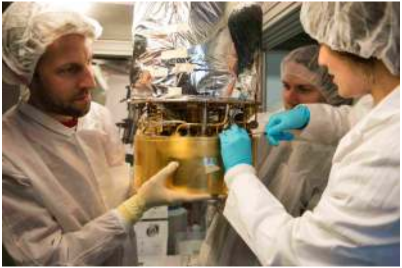
- Microwave frequency : $\nu(\text{TEM}_{900}) = 51.099\ \text{GHz}$
- Long lifetime $T_{\text{cav}}$ = some tens of milliseconds !
- Small mode volume $V \simeq 700\ \text{mm}^3$
- $\mathcal{E}_0 = \sqrt{\frac{\hbar\omega_c}{2\varepsilon_0 V}} \simeq 1.5\ \text{mV/m}$



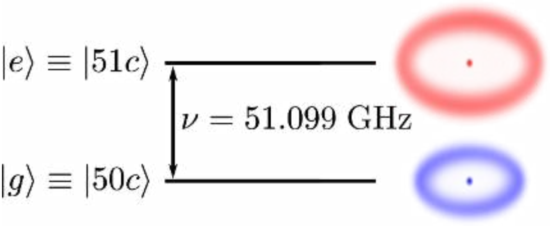


# Boxes to trap light



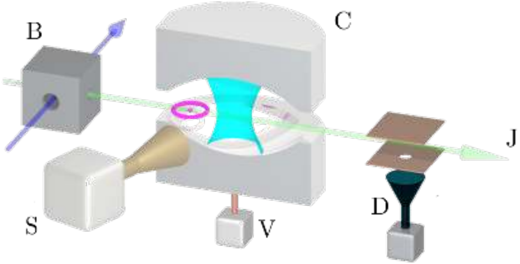


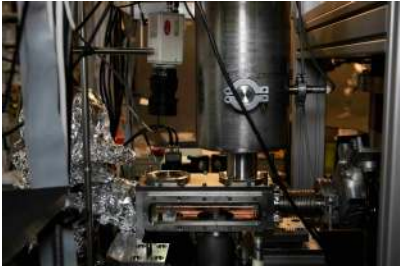
# Circular Rydberg atoms




- $^{85}$Rb
- Rydberg atoms : large principal quantum number $n \simeq 50$
- Circular Rydberg : maximal angular momentum : $l = |m| = n - 1$
- Long lifetime $T_{\text{at}} \simeq 30$ ms
- Microwave transitions
- Large dipole : $d = 1776|q|a_0$ for $|50c\rangle \rightarrow |51c\rangle$

# Preparing atomic samples



- Paquets with typically 0.1 atoms
- Velocity $250 \mathrm{~m/s}$
- Interaction with cavity controled by V
- Ionization detector D : detects $|e\rangle$ or $|g\rangle$

# State superposition

Goal : we want to understand a notation such as $|\pm_x\rangle = (|e\rangle \pm |g\rangle)/\sqrt{2}$.
Note that :

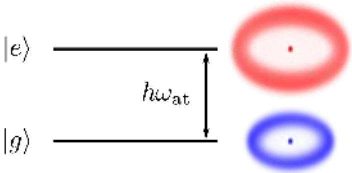
$$
\begin{aligned}
P_e &= \langle e|+_x\rangle = \frac{1}{2}, \\
P_g &= \langle g|+_x\rangle = \frac{1}{2}.
\end{aligned}
$$

But if we change the measurement basis :

$$
\begin{aligned}
P_+ &= \langle +_x|+_x\rangle = 1, \\
P_- &= \langle -_x|+_x\rangle = 0.
\end{aligned}
$$

What does it mean to change the measurement basis ?

# The Bloch sphere



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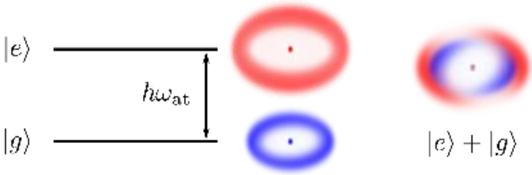

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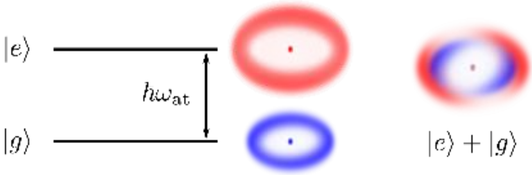
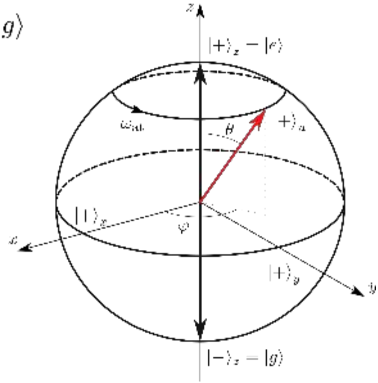



Most general state (up to a global phase) :

$$|\psi\rangle = \cos\theta\,|e\rangle + \sin\theta e^{i\varphi}\,|g\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta e^{i\varphi} \end{pmatrix}.$$



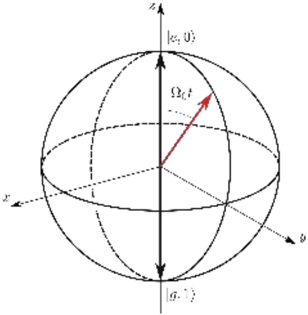
# Atom-cavity interaction : The Jaynes-Cummings Hamiltonian

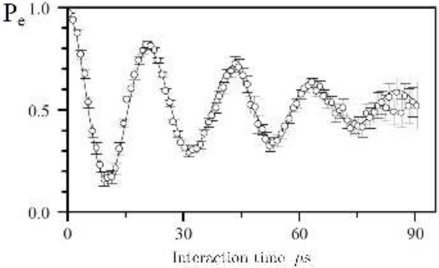
$$\hat{H}_{\mathrm{JC}} = \frac{\hbar\omega_{\mathrm{at}}}{2}\hat{\sigma}_z + \hbar\omega_c\left(\hat{N} + \frac{1}{2}\right) + \frac{\hbar\Omega_0}{2}\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-\right)$$

$$\Omega_0 = \frac{2d\mathcal{E}_0\vec{\epsilon}_a^{\,*}\cdot\vec{\epsilon}_c}{\hbar} \simeq 2\pi\cdot 50\ \mathrm{kHz} >> T_{\mathrm{at}}^{-1}, T_{\mathrm{cav}}^{-1}$$

# Interaction of one atom with one photon : Rabi oscillations

- $\hat{H}_{\text{jc}}$ commutes with the number of excitations and can be separated in terms $\hat{H}_n$ with $n$ excitations.
- For an atom initially in $|e\rangle$ with no photon, one can restrict to the subspace of states $\{|e,0\rangle, |g,1\rangle\}$.



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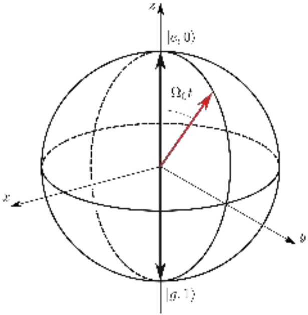


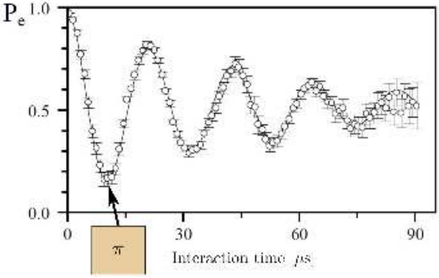

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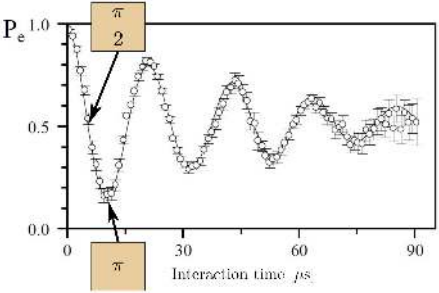
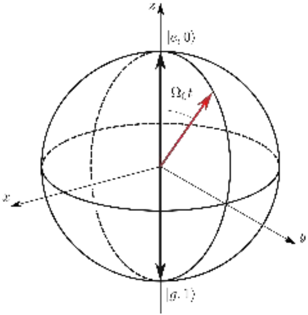
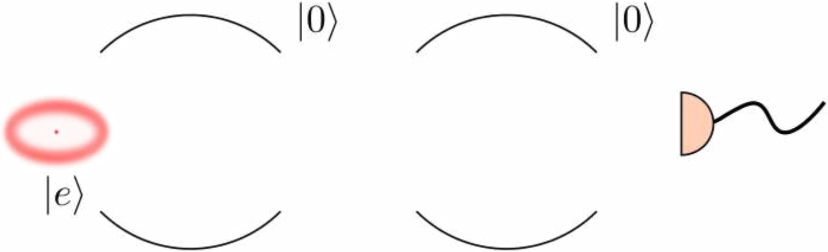


# Preparation of an entangled state



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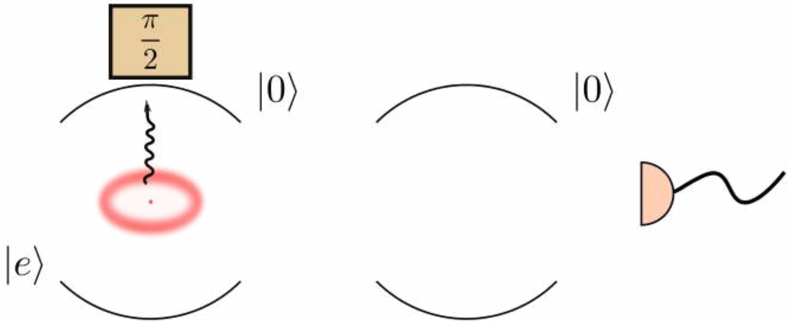

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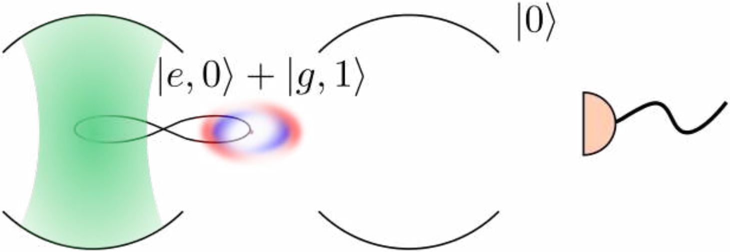

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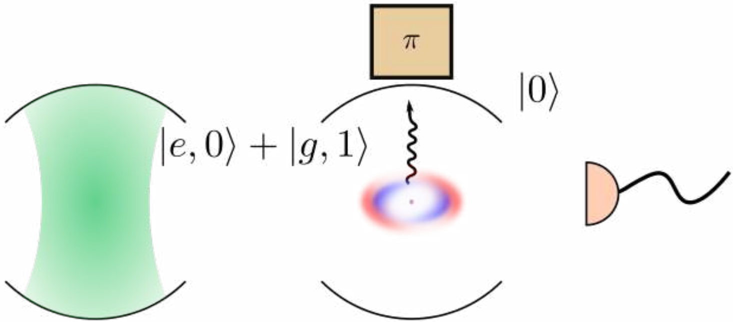

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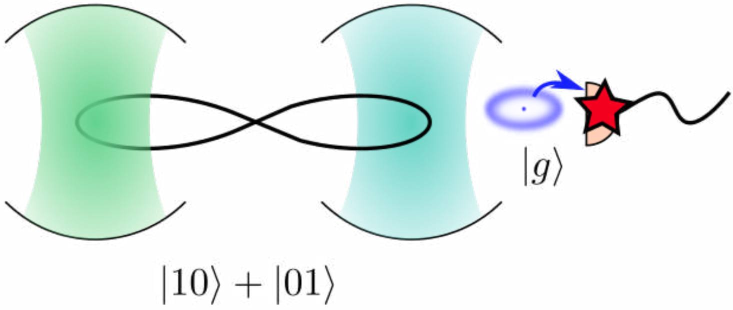

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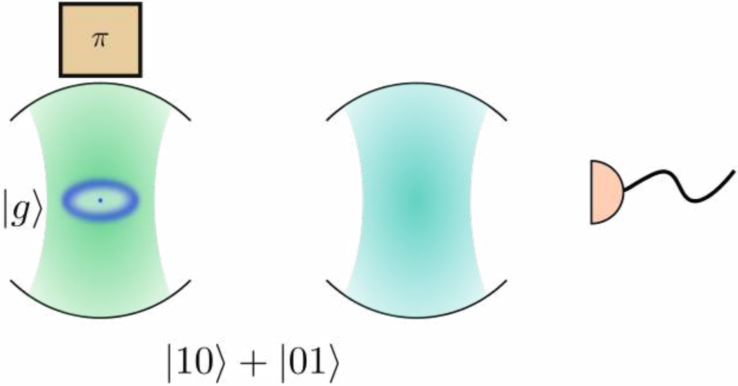

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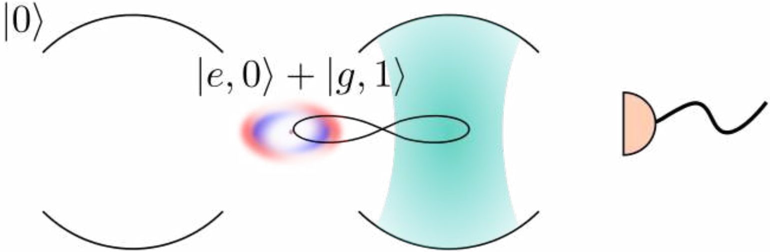

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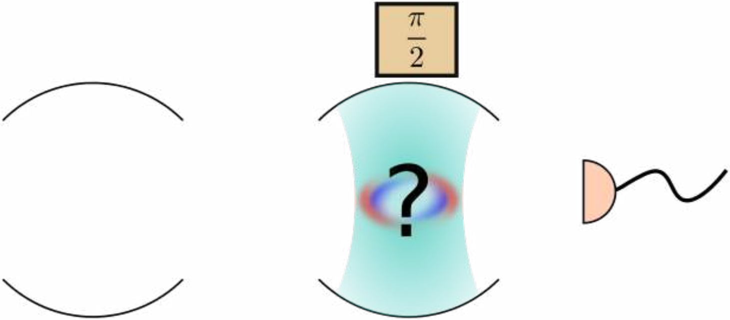
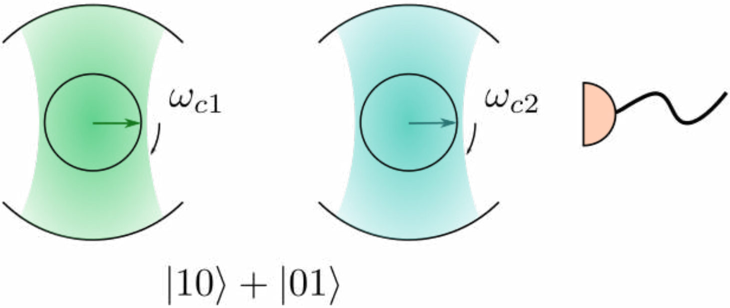


# The role of the quantum phase



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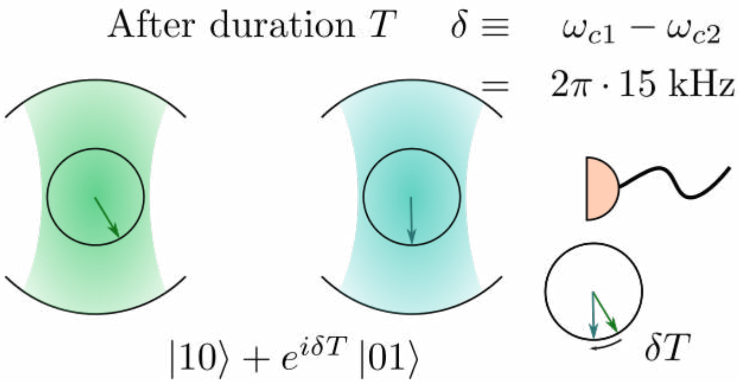
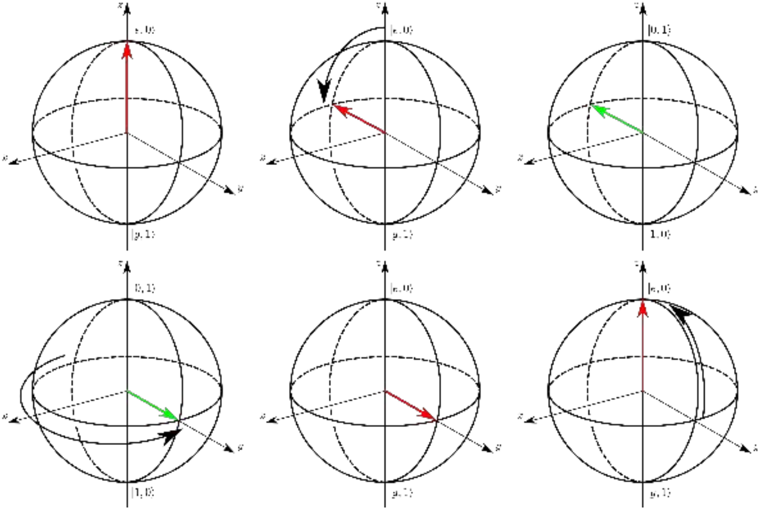
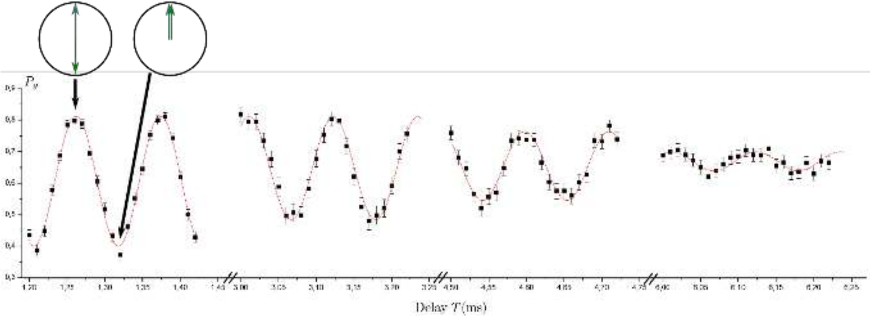


# To absorb or not to absorb

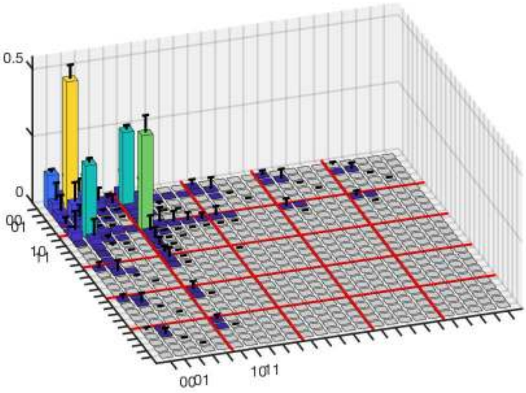


# Interferometric signal




- Signature of the non-local coherence between $|10\rangle$ and $|01\rangle$
- Signal reveals the frequency difference between the two cavities

# Reconstructed density matrix



- Preparation and probe of a state $|10\rangle + |01\rangle$, with tomography ;
- The non-local coherences play a major role in the sensitivity of the state ;
- The photon is sensitive to the frequency beat of the cavities : it's in both of them at the same time !

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- The non-local coherences play a major role in the sensitivity of the state ;
- The photon is sensitive to the frequency beat of the cavities : it's in both of them at the same time !
- Perpective : Preparation of $|20\rangle + |02\rangle$

Thank you for your attention !

# The density matrix in a nutshell

$$|e\rangle \to |e\rangle\langle e| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{array}{c} \begin{matrix} e & g \end{matrix} \\ \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} \end{array}$$

$$|+\rangle \to |+\rangle\langle +| = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

$$|\psi_\varphi\rangle \equiv (|e\rangle + e^{i\varphi}|g\rangle)/\sqrt{2} \to \frac{1}{2}\begin{pmatrix}1 & e^{-i\varphi}\\ e^{i\varphi} & 1\end{pmatrix}$$

Moere generally, every matrix that fullfills :

$$\begin{aligned} \mathrm{Tr}(\hat{\rho}) &= 1, \\ \hat{\rho}^\dagger &= \hat{\rho}, \\ \hat{\rho} &> 0, \end{aligned}$$

is a density matrix.

# Diagonalizing some density matrices

Diagonalization of $\hat{\rho}_\varphi \equiv |\psi_\varphi\rangle\langle\psi_\varphi| = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\varphi} \\ e^{i\varphi} & 1 \end{pmatrix}$ :

$$\begin{aligned} \hat{\rho}_\varphi |\psi_\varphi\rangle &= |\psi_\varphi\rangle \langle\psi_\varphi|\psi_\varphi\rangle &= |\psi_\varphi\rangle, \\ \hat{\rho}_\varphi |\psi_{-\varphi}\rangle &= |\psi_\varphi\rangle \langle\psi_\varphi|\psi_{-\varphi}\rangle &= 0. \end{aligned}$$

Hence,

$$\hat{\rho}_\varphi = \hat{P} \begin{array}{c} \begin{array}{cc} \psi_\varphi & \psi_{-\varphi} \end{array} \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{array} \hat{P}^\dagger.$$

Diagonalization of $\tilde{\hat{\rho}}_\varphi \equiv \frac{1}{2}\begin{pmatrix} 1 & ce^{-i\varphi} \\ ce^{i\varphi} & 1 \end{pmatrix} = \frac{1+c}{2}\hat{\rho}_\varphi + \frac{1-c}{2}\hat{\rho}_{-\varphi}$, where $(c \in [-1, 1])$ :

$$\tilde{\hat{\rho}}_\varphi = \hat{P} \begin{pmatrix} \frac{1+c}{2} & 0 \\ 0 & \frac{1-c}{2} \end{pmatrix} \hat{P}^\dagger.$$

- Born's rule :

$$p(m) = |\langle m|\psi\rangle|^2 \rightarrow p(m) = \langle m|\,\hat{\rho}\,|m\rangle$$

- Average :

$$\langle A\rangle = \mathrm{Tr}(\hat{\rho}\hat{A})$$

- Simple expression for $\hat{\rho}$ :

$$\hat{\rho} = \frac{1}{2}\left(\mathbb{1} + \vec{r}\cdot\hat{\vec{\sigma}}\right)$$